

## RESEARCH ARTICLE

# Power Allocation, Relay Selection and Energy Cooperation Strategies in Energy Harvesting Cooperative Wireless Networks

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## ABSTRACT

In this paper, optimal power allocation and relay selection strategies in energy harvesting cooperative wireless networks are studied. In particular, signal-to-noise ratio (SNR)-maximizing based power allocation and relay selection without and with energy cooperation—via wireless energy transfer—are considered. Moreover, total relay power minimization subject to target end-to-end SNR is investigated. The different optimal strategies are formulated as optimization problems, which are non-convex. Thus, intelligent transformations are applied to transform non-convex problems into convex ones, and polynomial-time solution procedures are proposed. Simulation results illustrate that power allocation strategies achieve higher end-to-end SNR than relay selection ones. Finally, energy cooperation is shown to be effective in improving end-to-end SNR, while total relay power minimization balances end-to-end SNR, transmit power consumption, and harvested energy.

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## KEYWORDS

Energy cooperation, energy harvesting, optimization, power allocation, relay selection, signal-to-noise ratio

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## 1. INTRODUCTION

Energy harvesting (EH) has recently appeared as a promising means to extend network life-time and minimize network maintenance costs, ultimately sustaining green wireless networks. Network nodes intermittently harvest energy from random energy sources from the surrounding environment for future use [1]. Due to the random nature of energy arrivals and the time-varying channel conditions, harvested energy and energy availability become random and cannot be predicted in advance. Moreover, stored energy in limited-capacity batteries depends not only on the amounts of harvested energy, but also on the previous signal transmissions. In turn, some network nodes may become energy deprived if harvested energy and transmit power are not cautiously managed [2]. In such scenarios, a balance between the achievable end-to-end signal-to-noise ratio (SNR) and power consumption is imperative, so as to guarantee quality-of-service (QoS). Additionally, energy cooperation—in the form of wireless energy transfer between network nodes—becomes highly desirable to improve

network performance. All these factors make power allocation and relay selection strategies in EH cooperative wireless networks more challenging than in conventional networks, requiring careful designs of such strategies.

Recently, several research works have studied power allocation and relay selection in EH wireless networks. For instance, in [3], the authors designed a low-complexity online step-wise and time-continuous transmit power policies with partial statistical knowledge of energy arrivals, where the optimal transmit-power decisions are performed after energy arrivals and are based on the amounts of harvested energy. The authors in [4] study energy harvesting and energy cooperation in a simple cooperative network composed of a source, a relay and a destination node. Specifically, both the source and the relay harvest energy from the surrounding environment and the source assists the relay by sharing some of its harvested energy. Moreover, the authors assume that the source and the relay nodes are fully informed about energy arrivals, and thus design offline energy management policies to maximize end-to-end throughput. In [5], the authors also study a simple cooperative network with EH source and decode-and-forward relay nodes for throughput maximization over a finite number of transmission intervals. Particularly, the authors proposed an offline convex optimization problem for power allocation; while for the online case, a dynamic programming approach is proposed for optimal transmit power. Additionally, suboptimal online schemes are proposed, providing tradeoff between complexity and performance. Joint power allocation and relay selection in EH amplify-and-forward (AF) systems is studied in [6]. Specifically, the authors proposed an optimal offline optimization problem and two sub-optimal low-complexity online power allocation schemes for throughput maximization. In [7], energy diversity via joint power assignment and relay selection is studied so as to maximize the minimum utility among all transmissions. Particularly, the authors proposed a suboptimal algorithm—based on statistical channel side information—that provides near-optimal performance and outperforms that of best-effort cooperation. In [8], the authors further study relay selection under non-causal or causal channel side information (CSI) and energy side information (ESI). With non-causal CSI/ESI, an offline relay selection problem is solved via a branch-and-bound algorithm, which serves as an upper-bound to the system performance. With causal CSI and non-causal/causal ESI, a dynamic programming problem is proposed, where a relay is selected if it has enough energy and results in the highest instantaneous throughput compared with the average throughput. In [9], the authors study the problems of throughput maximization by some deadline, and the minimization of the time delay to transmit a certain number of bits in fading channels. Particularly, the first problem is solved via deterministic (offline) and stochastic (online) policies; while the second problem is solved via a deterministic policy. In [10], offline throughput maximization for two-hop EH communication networks with non-causal energy arrivals is studied. Specifically, the authors study the cases of single relay and two relays, and provide convex formulations that yield optimal transmission policies. The authors in [11] study the problem of optimal energy allocation over a finite number of time-slots for throughput maximization, with time-varying channel conditions and harvested energy, and two types of side information (SI). Specifically, the cases of causal SI (i.e. past and present time-slots) and full SI (past, present, and future time-slots) are studied. The solutions for these cases are based on offline dynamic programming and convex optimization techniques. In [12], the minimization of transmission completion time for a certain number of bits per user over broadcast channels with known energy harvesting instants and packet arrival times is studied. Particularly, an iterative offline algorithm is utilized to schedule the transmit power levels and rates across time. The transmission completion time minimization problem has also been studied in [13] for EH broadcast channels. Particularly, the objective is to minimize the time by which all of the rechargeable transmitter's packets—assuming an infinite-sized battery—are delivered to their intended receivers. Moreover, an iterative algorithm is developed to obtain the globally optimal offline transmission policy, where the total transmit power is split optimally based on defined cut-off power levels. A similar problem is considered in [14], where the battery capacity is assumed to be finite. Furthermore, a directional water-filling algorithm is proposed to find the optimal shares of the users from the total power. The problem of optimal transmission powers and rates in multiple-access channels for the minimization

of transmission time with energy harvesting transmitters is studied in [15]. Particularly, an offline generalized iterative backward water-filling algorithm is developed, where the energy harvesting times and harvested energy amounts are assumed to be known. The solution of the developed algorithm is based on decomposing the transmission completion time into convex optimization problems. In [16], power allocation and energy transfer policies for optimal sum-rate maximization in a multi-source single-relay wireless network are studied. The authors assume the source nodes and the relay to harvest energy, with energy sharing capability. Moreover, the sum-rate maximization problem is decomposed into optimal energy transfer (OET) and optimal power allocation (OPA) problems. The solution to the OET problem reduces to an ordered node selection based on wireless and energy transfer channels; while that of the OPA problem reduces to a directional water-filling solution. In [17], the use of voluntary AF relays that participate in transmission only if they have sufficient energy is proposed. Additionally, the authors analyze the symbol error rate performance for energy constrained and unconstrained relays, and conclude that energy usage not only depends on transmit power setting, but also on the number of available relays.

In this paper, different power allocation and relay selection strategies are studied for multi-user cooperative wireless networks with multiple EH amplify-and-forward relays\*. Particularly, SNR-maximizing power allocation and relay selection strategies are considered, where each relay only utilizes its own harvested energy. After that, energy cooperation—via wireless energy transfer—is augmented with the different strategies to efficiently share and utilize the harvested energy among the relay nodes. On the other hand, it is intuitive that increasing the transmit power of a relay improves the end-to-end SNR; however, it may unnecessarily consume all its harvested energy, which otherwise could have been stored for later transmission of its data, or cooperation with other nodes. Therefore, total relay power minimization subject to target end-to-end SNR (which provides QoS guarantee) is also studied to achieve a balance between end-to-end SNR, transmit power consumption, and harvested energy. The different power allocation and relay selection strategies without and with energy cooperation are formulated as optimization problems, which happen to be non-convex, as they involve non-convexities in the objective function and/or constraints [18]. Consequently, such problems are difficult to solve in real-time, requiring computationally-intensive global optimization techniques. Therefore, intelligent transformations and techniques are applied to convert the original non-convex problems into convex and/or linear programming ones. After that, polynomial-time solution procedures are devised, which can be efficiently executed, yielding optimal solutions.

In contrast to previous works, this work provides an overall comparison of power allocation and relay selection strategies for end-to-end SNR maximization in multi-user multi-relay energy harvesting wireless networks, while taking into account energy cooperation. In addition, total relay power minimization subject to QoS guarantee is evaluated under different combinations of power allocation, relay selection and energy cooperation strategies. This has been achieved via novel optimization-theoretic formulations, and intelligent reformulations and solution procedures.

The main contributions of this paper are summarized as follows.

- Formulation of the optimal SNR-maximizing:
  - power allocation without and with energy cooperation problems as multi-objective linear fractional optimization problems, and reformulating them into parametric linear programming problems.
  - relay selection without and with energy cooperation problems as mixed integer nonlinear fractional programming problems, and reformulating them into integer fractional programs.
- Devising optimal polynomial-time solution procedures for the different power allocation and relay selection problems, without and with energy cooperation.

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\*It should be noted that the power allocation and relay selection strategies studied in this paper are online solutions, since energy arrivals and channel conditions are random and not known a-priori [5].

- Formulation of the optimal total relay power minimization subject to target end-to-end SNR:
  - without and with energy cooperation as linear programming problems, which can be solved efficiently using any standard optimization software package.
  - with relay selection, and joint relay selection and energy cooperation problems as mixed integer nonlinear programming problems, which—after appropriate transformation—can be solved using polynomial-time solution procedures.

This paper is organized as follows. In Section 2, the network model with energy harvesting relays is presented. Sections 3 and 4 discuss SNR-maximizing power allocation and relay selection strategies, respectively. Power allocation and relay selection strategies with energy cooperation are studied in Section 5. In Section 6, total relay power minimization subject to target end-to-end SNR is considered. Simulation results are presented in Section 7 to validate the different strategies and their solution procedures. Finally, conclusions are drawn in Section 8.

## 2. NETWORK MODEL

Consider a wireless network with  $N$  source-destination pairs, denoted  $S_i - D_i$  for  $i \in \{1, 2, \dots, N\}$ , and  $K$  EH amplify-and-forward relays  $R_k$ , for  $k \in \{1, 2, \dots, K\}$ . The source-relay  $S_i - R_k$  and relay-destination  $R_k - D_i$  channel coefficients  $h_{s_i, r_k}$  and  $h_{r_k, d_i}$ , respectively, are modeled as narrowband Rayleigh fading with additive white Gaussian noise (AWGN). Particularly, let  $h_{s_i, r_k} \sim \mathcal{CN}(0, \sigma_{s_i, r_k}^2)$  and  $h_{r_k, d_i} \sim \mathcal{CN}(0, \sigma_{r_k, d_i}^2)$ , where  $\sigma_{s_i, r_k}^2 = d_{s_i, r_k}^{-\nu}$  and  $\sigma_{r_k, d_i}^2 = d_{r_k, d_i}^{-\nu}$  are the channel variances, expressed in terms of the inter-node distances, and the path-loss exponent  $\nu$ . Additionally, each source-destination pair  $S_i - D_i$  is assigned a signature waveform  $c_i(t)$ , which allows multiuser detection at each destination node. The cross-correlation coefficient between waveforms  $c_i(t)$  and  $c_j(t)$  is denoted  $\rho_{i,j}$ , where  $0 \leq \rho_{i,j} \leq 1$  for  $i \neq j$ , and  $\rho_{i,i} = 1, \forall i \in \{1, 2, \dots, N\}$ . No direct link is assumed between each source and destination node. Furthermore, the power consumption for data reception and processing at each relay is assumed to be negligible. Without loss of generality, it is assumed that all source nodes have the same transmit power (i.e.  $P_{s_i} = P_s, \forall i \in \{1, 2, \dots, N\}$ ).

Communication between each source-destination pair is performed over two time-slotted phases: (1) broadcasting phase ( $N$  time-slots), and cooperation phase (1 multiple-access time-slot). Let  $\zeta \geq 1$  represent a communication frame, which consists of  $N + 1$  equal-length unit-duration time-slots<sup>†</sup>. For the first frame, each relay harvests energy from the  $N$  prior time-slots and utilizes the harvested energy for cooperative relaying in the cooperation phase (see Fig. 1). It should be noted that any harvested energy during the multiple-access time-slot of the cooperation phase is not utilized for cooperative transmission in that frame, but instead stored for use in the following communication frame(s). Hence, for the following frames (i.e.  $\zeta > 1$ ), each relay harvests energy from  $N + 1$  time-slots. Additionally, channels experience block-fading and remain constant during each communication frame, but change from one frame to another.

Energy arrivals at each relay  $R_k$  are modeled as a Poisson process with rate  $\lambda_{r_k}$  per time-slot [19]. Now, let  $\mathcal{N}_{r_k}(t(\zeta))$  be the number of energy arrivals up to and including the  $N^{th}$  time-slot  $t(\zeta) = N + (\zeta - 1) \cdot (N + 1)$  of frame  $\zeta$ , where  $\zeta \geq 1$  and  $\mathcal{N}_{r_k}(t(\zeta)) \geq \mathcal{N}_{r_k}(t(\zeta'))$ ,  $\forall \zeta \geq \zeta' > 0$ . Also, let  $\tau_{r_k, m}$  (for  $m = 0, 1, \dots$ ) be the random time instants of energy arrivals at relay  $R_k$ , with  $\tau_{r_k, m'} < \tau_{r_k, m}$  for  $m' < m$  and  $\tau_{r_k, 0} = 0, \forall k \in \{1, 2, \dots, K\}$ . In addition, let the energy arrivals  $E_{r_k}(\tau_{r_k, m})$  be modeled as i.i.d. uniform random variables, such that  $E_{r_k}(\tau_{r_k, m}) \sim U(0, E_{\max, r_k})$ , where  $E_{\max, r_k}$  is the maximum value of an energy arrival at relay  $R_k$ . Moreover, let  $E_{\Sigma, r_k}^{\zeta}$  be the total harvested energy up to the  $N^{th}$  time-slot of communication frame  $\zeta$ , such that<sup>‡</sup>

<sup>†</sup> Due to the normalized time-slots, power and energy terms can be used interchangeably.

<sup>‡</sup> Depending on the energy source, energy arrivals are usually random, sporadic in nature, and characterized with small amounts (i.e. energy arrivals are only known causally) [17]. This corresponds to the case where energy is harvested from RF signals at the relays; only when such random signals are received with sufficient strength.

$$E_{\Sigma, r_k}^{\zeta} = \sum_{m=1}^{\mathcal{N}_{r_k}(t(\zeta))} E_{r_k}(\tau_{r_k, m}) \quad (1)$$

The amount of harvested energy during communication frame  $\zeta$  is given by  $\Delta E_{\Sigma, r_k}^{\zeta} = E_{\Sigma, r_k}^{\zeta} - E_{\Sigma, r_k}^{\zeta-1}$ , for  $\zeta \geq 2$ . Assuming battery-capacity constraint  $B_{\max}$  at all relays, then  $E_{\Sigma, r_k}^{\zeta} \leq B_{\max}$ .

**Remark 1:** The total harvested energy in any communication frame  $\zeta$  must satisfy  $E_{\Sigma, r_k}^{\zeta} = \min(\Delta E_{\Sigma, r_k}^{\zeta} + E_{\Sigma, r_k}^{\zeta-1}, B_{\max})$ ,  $\forall k \in \{1, 2, \dots, K\}$ .

## 2.1. Broadcasting Phase

In the broadcasting phase of communication frame  $\zeta$ , the received signal at relay  $R_k$  from source node  $S_i$  is given by

$$y_{s_i, r_k}^{\zeta} = \sqrt{P_s} h_{s_i, r_k}^{\zeta} x_i^{\zeta} + n_{s_i, r_k}^{\zeta}, \quad (2)$$

where  $x_i^{\zeta}$  is the source node's symbol, and  $n_{s_i, r_k}^{\zeta}$  is the zero-mean  $N_0$ -variance AWGN sample at relay  $R_k$ . The SNR of the detected received signal is obtained as [20]

$$\gamma_{s_i, r_k}^{\zeta} = \frac{P_s |h_{s_i, r_k}^{\zeta}|^2}{N_0}. \quad (3)$$

At the end of the broadcasting phase, each relay  $R_k$  will have received  $N$  signals  $\{y_{s_i, r_k}^{\zeta}\}_{i=1}^N$  from the  $N$  source nodes.

## 2.2. Cooperation Phase

In the multiple-access time-slot of the cooperation phase of frame  $\zeta$ , each relay  $R_k$  simultaneously transmits the following signal

$$\mathcal{X}_{r_k}^{\zeta}(t) = \sum_{j=1}^N \beta_{j,k}^{\zeta} y_{s_j, r_k}^{\zeta} c_j(t), \quad (4)$$

where  $\beta_{j,k}^{\zeta} = \sqrt{\frac{1}{P_s |h_{s_j, r_k}^{\zeta}|^2 + N_0}}$  is a normalization factor. The received signal at destination node  $D_i$  is written as<sup>§</sup>

$$y_{d_i}^{\zeta}(t) = \sum_{k=1}^K \sqrt{P_{r_k}^{\zeta}} h_{r_k, d_i}^{\zeta} \mathcal{X}_{r_k}^{\zeta}(t) + n_{d_i}^{\zeta}(t), \quad (5)$$

where  $P_{r_k}^{\zeta}$  is the cooperative transmit power allocated by relay  $R_k$  in communication frame  $\zeta$ , and  $n_{d_i}^{\zeta}(t)$  is the AWGN process at destination node  $D_i$ . Substituting (2) and (4) into (5) gives

$$y_{d_i}^{\zeta}(t) = \sum_{k=1}^K \sum_{j=1}^N \beta_{j,k}^{\zeta} \sqrt{P_{r_k}^{\zeta}} h_{s_j, r_k}^{\zeta} h_{r_k, d_i}^{\zeta} x_j^{\zeta} c_j(t) + \bar{n}_{d_i}^{\zeta}(t), \quad (6)$$

where

$$\bar{n}_{d_i}^{\zeta}(t) = n_{d_i}^{\zeta}(t) + \sum_{k=1}^K \sqrt{P_{r_k}^{\zeta}} h_{r_k, d_i}^{\zeta} \sum_{j=1}^N \beta_{j,k}^{\zeta} y_{s_j, r_k}^{\zeta} c_j(t). \quad (7)$$

Let  $\rho_{i,j} = \rho$ ,  $\forall i \neq j$ , then the decorrelated received signal of source node  $S_i$  at destination node  $D_i$  is obtained as [20]

$$y_{d_i}^{\zeta} = \sum_{k=1}^K \left( \beta_{i,k}^{\zeta} \sqrt{P_{r_k}^{\zeta}} h_{s_i, r_k}^{\zeta} h_{r_k, d_i}^{\zeta} \right) x_i^{\zeta} + \bar{n}_{d_i}^{\zeta}, \quad (8)$$

where  $\bar{n}_{d_i}^{\zeta} \sim \mathcal{CN}\left(0, N_0 \varrho_N \left( \sum_{k=1}^K P_{r_k}^{\zeta} \left( \beta_{i,k}^{\zeta} \right)^2 |h_{r_k, d_i}^{\zeta}|^2 + 1 \right) \right)$ , and  $\varrho_N$  is given by

<sup>§</sup> Perfect timing synchronization is assumed among the relay nodes.

$$\varrho_N = \frac{1 + (N-2)\rho}{1 + (N-2)\rho - (N-1)\rho^2}. \quad (9)$$

The instantaneous SNR of the received decorrelated signal  $y_{d_i}^\zeta$  can be shown to be

$$\gamma_{s_i, d_i}^\zeta = \frac{1}{N_0 \varrho_N} \frac{\sum_{k=1}^K P_s P_{r_k}^\zeta (\beta_{i,k}^\zeta)^2 |h_{s_i, r_k}^\zeta|^2 |h_{r_k, d_i}^\zeta|^2}{\sum_{k=1}^K P_{r_k}^\zeta (\beta_{i,k}^\zeta)^2 |h_{r_k, d_i}^\zeta|^2 + 1}, \quad (10)$$

which can be re-expressed as

$$\gamma_{s_i, d_i}^\zeta = \frac{\sum_{k=1}^K P_{r_k}^\zeta \xi_{i,k}^\zeta}{\sum_{k=1}^K P_{r_k}^\zeta \chi_{k,i}^\zeta + 1}, \quad (11)$$

where  $\xi_{i,k}^\zeta = \frac{P_s (\beta_{i,k}^\zeta)^2 |h_{s_i, r_k}^\zeta|^2 |h_{r_k, d_i}^\zeta|^2}{N_0 \varrho_N}$ , and  $\chi_{k,i}^\zeta = (\beta_{i,k}^\zeta)^2 |h_{r_k, d_i}^\zeta|^2$ . For convenience, let  $\boldsymbol{\xi}_i^\zeta = [\xi_{i,1}^\zeta, \xi_{i,2}^\zeta, \dots, \xi_{i,K}^\zeta]$ ,  $\boldsymbol{\chi}_i^\zeta = [\chi_{1,i}^\zeta, \chi_{2,i}^\zeta, \dots, \chi_{K,i}^\zeta]$ , and  $\mathbf{p}_r^\zeta = [P_{r_1}^\zeta, P_{r_2}^\zeta, \dots, P_{r_K}^\zeta]^T$ , where  $[\cdot]^T$  denotes transposition. Hence,  $\gamma_{s_i, d_i}^\zeta$  is re-written as

$$\gamma_{s_i, d_i}^\zeta(\mathbf{p}_r^\zeta) \triangleq \frac{\boldsymbol{\xi}_i^\zeta \mathbf{p}_r^\zeta}{\boldsymbol{\chi}_i^\zeta \mathbf{p}_r^\zeta + 1}, \quad (12)$$

which is a ratio of two linear functions.

**Remark 2:** The SNR function  $\gamma_{s_i, d_i}^\zeta(\mathbf{p}_r^\zeta)$ ,  $\forall i \in \{1, 2, \dots, N\}$  is a linear fractional (LF) function that is quasi-monotonic (i.e. both pseudo-convex and pseudo-concave) in  $P_{r_k}^\zeta$ ,  $\forall k \in \{1, 2, \dots, K\}$ , with  $\boldsymbol{\chi}_i^\zeta \mathbf{p}_r^\zeta + 1 > 0$  [21].

It should be noted that there is a total power constraint per time-slot  $P_{\max}$ , such that  $P_s \leq P_{\max}$ , and  $P_{r_k}^\zeta \leq P_{\max}$ ,  $\forall k \in \{1, 2, \dots, K\}$ . In turn, the total transmit relay power must satisfy  $\sum_{k=1}^K P_{r_k}^\zeta \leq P_{\max}$ ,  $\forall \zeta \geq 1$ .

**Remark 3:** In any communication frame  $\zeta$ ,  $P_{r_k}^\zeta \leq \min \{E_{\Sigma, r_k}^\zeta, P_{\max}\} \triangleq \bar{E}_{\Sigma, r_k}^\zeta$ ,  $\forall k \in \{1, 2, \dots, K\}$ .

### 3. POWER ALLOCATION

The aim is to optimally allocate the harvested energy such that the SNR of each source-destination pair is maximized in each communication frame  $\zeta$ . Hence, the optimal SNR-maximizing power allocation (OPT-PA) problem is formulated as a multi-objective linear fractional optimization problem as<sup>¶</sup>

**OPT-PA ( $\zeta$ ):**

$$\max \left( \frac{\boldsymbol{\xi}_1^\zeta \mathbf{p}_r^\zeta}{\boldsymbol{\chi}_1^\zeta \mathbf{p}_r^\zeta + 1}, \frac{\boldsymbol{\xi}_2^\zeta \mathbf{p}_r^\zeta}{\boldsymbol{\chi}_2^\zeta \mathbf{p}_r^\zeta + 1}, \dots, \frac{\boldsymbol{\xi}_N^\zeta \mathbf{p}_r^\zeta}{\boldsymbol{\chi}_N^\zeta \mathbf{p}_r^\zeta + 1} \right)$$

$$\text{s.t.} \quad \sum_{k=1}^K P_{r_k}^\zeta \leq P_{\max}, \quad (13a)$$

$$0 \leq P_{r_k}^\zeta \leq \bar{E}_{\Sigma, r_k}^\zeta, \quad \forall k \in \{1, 2, \dots, K\}. \quad (13b)$$

The first constraint ensures that the total relay transmit power does not exceed the transmit power constraint  $P_{\max}$ , while the second constraint ensures that the transmitted power per relay  $R_k$  does not exceed its total harvested energy.

The **OPT-PA** ( $\zeta$ ) problem is generally non-convex, since the SNR function  $\gamma_{s_i, d_i}^\zeta(\mathbf{p}_r^\zeta)$  of each source-destination pair  $S_i - D_i$  is quasi-monotonic (see Remark 2); although the feasible region is a nonempty, compact convex set of linear constraints [22]. Moreover, the maximization of LF functions is known to be NP-complete [23]. Consequently, solving such problem for each communication frame  $\zeta$  is computationally prohibitive.

<sup>¶</sup>When a relay  $R_k$  transmits in the cooperation phase, it does so at a constant power  $P_{r_k}^\zeta$ .

To efficiently solve problem **OPT-PA** ( $\zeta$ ), one must maximize the LF function  $\gamma_{S_i, D_i}^\zeta(\mathbf{p}_r^\zeta)$  of each source-destination pair  $S_i - D_i$ , subject to the same set of constraints, i.e.

**LF-PA<sub>i</sub>** ( $\zeta$ ):

$$\max \frac{\xi_i^\zeta \mathbf{p}_r^\zeta}{\chi_i^\zeta \mathbf{p}_r^\zeta + 1}$$

$$\text{s.t. } \mathbf{A} \mathbf{p}_r^\zeta \leq \mathbf{b}^\zeta, \quad (14a)$$

$$\mathbf{p}_r^\zeta \geq \mathbf{0}, \quad (14b)$$

where  $\mathbf{A}$  is a  $(K+1) \times K$  matrix,  $\mathbf{b}^\zeta$  is a  $(K+1) \times 1$  vector, and  $\mathbf{0}$  is a  $K \times 1$  all-zeros vector. The **LF-PA<sub>i</sub>** ( $\zeta$ ) problem possesses some interesting properties; most importantly, a local maximum is also a global maximum, which is attained at an extreme point (i.e. a vertex) in the convex feasible region [24]. This leads to the following proposition [22].

**Proposition 1:** The LF function  $\gamma_{S_i, D_i}^\zeta(\mathbf{p}_r^\zeta)$  of source-destination  $S_i - D_i$  is maximized when only one of the relays sets  $P_{r_k}^\zeta = \bar{E}_{\Sigma, r_k}^\zeta$ , yielding the maximum value of  $\frac{\xi_i^\zeta \mathbf{p}_r^\zeta}{\chi_i^\zeta \mathbf{p}_r^\zeta + 1}$ , while the other relays are nulled (i.e.  $P_{r_l}^\zeta = 0, \forall l \neq k$ ).

**Proof:** By applying a solution of the Karush-Kuhn-Tucker optimality conditions, as in [25].  $\square$

Consequently, the solution to **LF-PA<sub>i</sub>** ( $\zeta$ ) is

$$\mathcal{Z}_i^\zeta \triangleq \max_{k \in \{1, 2, \dots, K\}} \frac{\bar{E}_{\Sigma, r_k}^\zeta \xi_{i, k}^\zeta}{\bar{E}_{\Sigma, r_k}^\zeta \chi_{k, i}^\zeta + 1}. \quad (15)$$

Based on the obtained solutions  $\mathcal{Z}_i^\zeta, \forall i \in \{1, 2, \dots, N\}$ , problem **OPT-PA** ( $\zeta$ ) is reformulated into a parametric linear programming (LP) problem, as [26]

**R-OPT-PA** ( $\zeta$ ):

$$\max \sum_{i=1}^N \left( \xi_i^\zeta \mathbf{p}_r^\zeta - \mathcal{Z}_i^\zeta \cdot \left( \chi_i^\zeta \mathbf{p}_r^\zeta + 1 \right) \right)$$

$$\text{s.t. } \mathbf{A} \mathbf{p}_r^\zeta \leq \mathbf{b}^\zeta, \quad (16a)$$

$$\mathbf{p}_r^\zeta \geq \mathbf{0}, \quad (16b)$$

which can be easily solved using any standard optimization software package. The solution procedure for optimal SNR-maximizing power allocation is summarized in Table I.

**Remark 4:** Under the **R-OPT-PA** ( $\zeta$ ) problem, not all the harvested energy is necessarily utilized (i.e.  $P_{r_k}^\zeta \leq \bar{E}_{\Sigma, r_k}^\zeta, \forall k \in \{1, 2, \dots, K\}$ ). Thus, in the following communication frame, the harvested energy is given by  $E_{\Sigma, r_k}^{\zeta+1} = \min \left( \Delta E_{\Sigma, r_k}^{\zeta+1} + E_{\Sigma, r_k}^\zeta - P_{r_k}^\zeta, B_{\max} \right)$ .

## 4. RELAY SELECTION

Relay selection (RS) can be implemented so as to reduce the number of simultaneous transmissions during the multiple-access time-slot of the cooperation phase. In turn, this alleviates the need for the stringent timing synchronization of all  $K$  relay nodes. It should be noted that relay selection in EH cooperative networks is slightly more involved than in conventional cooperative networks, as it not only depends on channel conditions, but also on the amount of harvested energy. For instance, a relay with the best source-relay and relay-destination channel conditions may have harvested the least amount of energy. Consequently, it may not necessarily be the optimal relay to select.

To formulate the SNR-maximizing relay selection problem, let  $\mathcal{I}_{r_k}^\zeta \in \{0, 1\}$ , for  $k \in \{1, 2, \dots, K\}$ , be a binary decision variable, where  $\mathcal{I}_{r_k}^\zeta = 1$  indicates that relay  $R_k$  is selected, while  $\mathcal{I}_{r_k}^\zeta = 0$ , otherwise. Consequently, the SNR function  $\gamma_{S_i, D_i}^\zeta$  of source-destination pair  $S_i - D_i$  in (11) can be written as

$$\bar{\gamma}_{s_i, d_i}^\zeta \triangleq \frac{\sum_{k=1}^K P_{r_k}^\zeta \xi_{i,k}^\zeta \cdot \mathcal{I}_{r_k}^\zeta}{\sum_{k=1}^K P_{r_k}^\zeta \chi_{k,i}^\zeta \cdot \mathcal{I}_{r_k}^\zeta + 1}. \quad (17)$$

Hence, the optimal SNR-maximizing relay selection (OPT-RS) problem can be formulated as

**OPT-RS ( $\zeta$ ):**

$$\max \prod_{i=1}^N \frac{\sum_{k=1}^K P_{r_k}^\zeta \xi_{i,k}^\zeta \cdot \mathcal{I}_{r_k}^\zeta}{\sum_{k=1}^K P_{r_k}^\zeta \chi_{k,i}^\zeta \cdot \mathcal{I}_{r_k}^\zeta + 1}$$

$$\text{s.t.} \quad \sum_{k=1}^K P_{r_k}^\zeta \cdot \mathcal{I}_{r_k}^\zeta \leq P_{\max}, \quad (18a)$$

$$\sum_{k=1}^K \mathcal{I}_{r_k}^\zeta = 1, \quad (18b)$$

$$0 \leq P_{r_k}^\zeta \leq E_{\Sigma, r_k}^\zeta \cdot \mathcal{I}_{r_k}^\zeta, \quad \forall k \in \{1, 2, \dots, K\}, \quad (18c)$$

$$\mathcal{I}_{r_k}^\zeta \in \{0, 1\}, \quad \forall k \in \{1, 2, \dots, K\}. \quad (18d)$$

The second constraint enforces the selection of the optimal relay, while the third constraint ensures that if a relay is not selected, then its transmit power is set to zero. Clearly, the **OPT-RS** ( $\zeta$ ) problem is a mixed integer nonlinear fractional programming problem, which belongs to the NP-hard class of mixed-integer nonlinear programming (MINLP) problems [27]. Therefore, such a problem is extremely computationally intensive, and a reformulation is necessary. As per Proposition 1, the LF function  $\bar{\gamma}_{s_i, d_i}^\zeta$  is maximized when  $P_{r_k}^\zeta = \bar{E}_{\Sigma, r_k}^\zeta$  for only one relay  $R_k$ ; while the other relays' transmit powers are set to zero. Thus, set  $P_{r_k}^\zeta \xi_{i,k}^\zeta \cdot \mathcal{I}_{r_k}^\zeta = \bar{E}_{\Sigma, r_k}^\zeta \xi_{i,k}^\zeta \cdot \mathcal{I}_{r_k}^\zeta \triangleq \bar{\xi}_{i,k}^\zeta \cdot \mathcal{I}_{r_k}^\zeta$ , and  $P_{r_k}^\zeta \chi_{k,i}^\zeta \cdot \mathcal{I}_{r_k}^\zeta = \bar{E}_{\Sigma, r_k}^\zeta \chi_{k,i}^\zeta \cdot \mathcal{I}_{r_k}^\zeta \triangleq \bar{\chi}_{i,k}^\zeta \cdot \mathcal{I}_{r_k}^\zeta$ . Also, note that the first constraint in problem **OPT-RS** ( $\zeta$ ) becomes  $\sum_{k=1}^K \bar{E}_{\Sigma, r_k}^\zeta \cdot \mathcal{I}_{r_k}^\zeta \leq P_{\max}$ , which is always satisfied as per Remark 3; and hence can be eliminated. In turn, problem **OPT-RS** ( $\zeta$ ) can be reformulated into an integer linear fractional program (ILFP) as follows

**R-OPT-RS ( $\zeta$ ):**

$$\max \prod_{i=1}^N \frac{\sum_{k=1}^K \bar{\xi}_{i,k}^\zeta \cdot \mathcal{I}_{r_k}^\zeta}{\sum_{k=1}^K \bar{\chi}_{i,k}^\zeta \cdot \mathcal{I}_{r_k}^\zeta + 1}$$

$$\text{s.t.} \quad \sum_{k=1}^K \mathcal{I}_{r_k}^\zeta = 1, \quad (19a)$$

$$\mathcal{I}_{r_k}^\zeta \in \{0, 1\}, \quad \forall k \in \{1, 2, \dots, K\}. \quad (19b)$$

The optimization of an ILFP is known to be NP-hard [28]. However, based on Proposition 1, problem **R-OPT-RS** ( $\zeta$ ) reduces to selecting relay  $R_k$  that maximizes the objective function when it sets  $P_{r_k}^\zeta = \bar{E}_{\Sigma, r_k}^\zeta$ , while all the other relays are idle. Hence, optimal relay selection  $R_{\text{OPT}}$  can be achieved by the solution procedure outlined in Table II.

**Remark 5:** The relays that have not been selected in communication frame  $\zeta$  remain in an idle state until the following communication frame. Additionally, the selected relay (say  $R_k$ ) utilizes  $P_{r_k}^\zeta = \bar{E}_{\Sigma, r_k}^\zeta$  for relaying; while the idle relays potentially get to harvest and store more energy. Hence, a relay with good channel conditions that was not selected in communications frame  $\zeta$  may have harvested high enough energy to be selected in the following communication frame.

## 5. ENERGY COOPERATION

Relay nodes may assist each other by sharing a portion of their harvested energy via a separate energy transfer unit, to efficiently utilize the harvested energy. However, wireless energy transfer is subject to an energy efficiency transfer



factor [4], and since there is a loss associated with such wireless transfer, a tradeoff between transferring energy or using it for relaying exists. Thus, this section considers the power allocation and relay selection strategies when augmented with energy cooperation.

### 5.1. Power Allocation

Let  $\varepsilon_{k,l}^\zeta \geq 0$  be the amount of energy transferred from relay  $R_k$  to relay  $R_l$ , for  $k \neq l$ . Also, let  $0 \leq \delta_{k,l} \leq 1$  be the end-to-end transfer efficiency factor<sup>||</sup>. In this work, the transfer energy factor is defined in terms of the distance between the relays, such that  $\delta_{k,l} = e^{-\eta d_{r_k,r_l}^2} = \delta_{l,k}$ , where  $\eta$  is a constant loss factor. Therefore, the farther apart the relays, the less the transfer efficiency. In turn, the total transmit power available at relay  $R_k$  becomes

$$0 \leq P_{r_k}^\zeta + \sum_{l=1, l \neq k}^K \delta_{l,k} \varepsilon_{l,k}^\zeta \leq P_{\max}, \quad (20)$$

while the transmit power and transferred energy must satisfy

$$0 \leq P_{r_k}^\zeta + \sum_{l=1, l \neq k}^K \varepsilon_{k,l}^\zeta \leq E_{\Sigma, r_k}^\zeta. \quad (21)$$

The wirelessly transferred energy  $\sum_{l=1, l \neq k}^K \delta_{l,k} \varepsilon_{l,k}^\zeta$  may temporarily be stored in a super-capacitor until cooperative transmission occurs [29]. This particularly happens when  $B_{\max} \leq P_{r_k}^\zeta + \sum_{l=1, l \neq k}^K \delta_{l,k} \varepsilon_{l,k}^\zeta \leq P_{\max}$ . In the case of  $P_{\max} \leq B_{\max}$ , a super-capacitor is not needed.

The SNR function  $\gamma_{s_i, d_i}^\zeta$  of source-destination pair  $S_i - D_i$  in (11) is re-expressed as

$$\bar{\gamma}_{s_i, d_i}^\zeta \triangleq \frac{\sum_{k=1}^K (P_{r_k}^\zeta + \sum_{l=1, l \neq k}^K \delta_{l,k} \varepsilon_{l,k}^\zeta) \xi_{i,k}^\zeta}{\sum_{k=1}^K (P_{r_k}^\zeta + \sum_{l=1, l \neq k}^K \delta_{l,k} \varepsilon_{l,k}^\zeta) \chi_{k,i}^\zeta + 1}. \quad (22)$$

The optimal SNR-maximizing joint power allocation and energy cooperation (OPT-PA-EC) problem is formulated as

#### OPT-PA-EC ( $\zeta$ ):

$$\begin{aligned} \max \quad & \left( \bar{\gamma}_{s_1, d_1}^\zeta, \bar{\gamma}_{s_2, d_2}^\zeta, \dots, \bar{\gamma}_{s_N, d_N}^\zeta \right) \\ \text{s.t.} \quad & \sum_{k=1}^K \left( P_{r_k}^\zeta + \sum_{l=1, l \neq k}^K \delta_{l,k} \varepsilon_{l,k}^\zeta \right) \leq P_{\max}, \end{aligned} \quad (23a)$$

$$0 \leq P_{r_k}^\zeta + \sum_{l=1, l \neq k}^K \varepsilon_{k,l}^\zeta \leq E_{\Sigma, r_k}^\zeta, \quad \forall k \in \{1, 2, \dots, K\}, \quad (23b)$$

$$P_{r_k}^\zeta, \varepsilon_{k,l}^\zeta \geq 0, \quad \forall k, l \in \{1, 2, \dots, K\} \text{ and } k \neq l. \quad (23c)$$

The first constraint ensures that the total transmit power constraint is satisfied, while the second constraint ensures that the total transmit relay power and transferred energy by a relay  $R_k$  does not exceed  $E_{\Sigma, r_k}^\zeta$ . The last constraint defines the range of values  $P_{r_k}^\zeta$  and  $\varepsilon_{k,l}^\zeta$  can take. As before, problem **OPT-PA-EC** ( $\zeta$ ) is non-convex. Thus, by the same token of problem **R-OPT-PA** ( $\zeta$ ), the above problem can be reformulated as a parametric LP problem as

#### R-OPT-PA-EC ( $\zeta$ ):

$$\begin{aligned} \max \quad & \sum_{i=1}^N \mathcal{S}_i^\zeta \\ \text{s.t.} \quad & \text{Constraints (23a) - (23c)}, \end{aligned} \quad (24)$$

where  $\mathcal{S}_i^\zeta$  is defined as

<sup>||</sup>The delay associated with energy transfer is assumed to be negligible.

$$S_i^\zeta \triangleq \sum_{k=1}^K \left( P_{r_k}^\zeta + \sum_{l=1, l \neq k}^K \delta_{l,k} \varepsilon_{l,k}^\zeta \right) \xi_{i,k}^\zeta - \bar{Z}_i^\zeta \cdot \left( \sum_{k=1}^K \left( P_{r_k}^\zeta + \sum_{l=1, l \neq k}^K \delta_{l,k} \varepsilon_{l,k}^\zeta \right) \chi_{k,i}^\zeta + 1 \right). \quad (25)$$

Additionally,  $\bar{Z}_i^\zeta$  is obtained as

$$\bar{Z}_i^\zeta \triangleq \max_{k \in \{1, 2, \dots, K\}} \frac{\bar{P}_{r_k}^\zeta \xi_{i,k}^\zeta}{\bar{P}_{r_k}^\zeta \chi_{k,i}^\zeta + 1}, \quad (26)$$

where  $\bar{P}_{r_k}^\zeta$  is defined as

$$\bar{P}_{r_k}^\zeta \triangleq \min \left\{ \bar{E}_{\Sigma, r_k}^\zeta + \sum_{l=1, l \neq k}^K \delta_{l,k} \varepsilon_{l,k}^\zeta, P_{\max} \right\}. \quad (27)$$

**Remark 6:** If  $E_{\Sigma, r_k}^\zeta \geq P_{\max}$  (and hence  $\bar{E}_{\Sigma, r_k}^\zeta = P_{\max}$ ), then relay  $R_k$  does not need to receive any energy from other relays; otherwise,  $\bar{E}_{\Sigma, r_k}^\zeta = E_{\Sigma, r_k}^\zeta$  and it must receive  $\sum_{l=1, l \neq k}^K \delta_{l,k} \varepsilon_{l,k}^\zeta$  to potentially reach  $P_{\max}$ .

Problem **R-OPT-PA-EC** ( $\zeta$ ) can be solved via the solution procedure provided in Table III.

## 5.2. Joint Relay Selection and Energy Cooperation

The optimal SNR-maximizing joint relay selection and energy cooperation (OPT-RS-EC) problem is formulated as

**OPT-RS-EC** ( $\zeta$ ):

$$\begin{aligned} \max \quad & \prod_{i=1}^N \frac{\sum_{k=1}^K (P_{r_k}^\zeta + \sum_{l=1, l \neq k}^K \delta_{l,k} \varepsilon_{l,k}^\zeta) \xi_{i,k}^\zeta \cdot \mathcal{I}_{r_k}^\zeta}{\sum_{k=1}^K (P_{r_k}^\zeta + \sum_{l=1, l \neq k}^K \delta_{l,k} \varepsilon_{l,k}^\zeta) \chi_{k,i}^\zeta \cdot \mathcal{I}_{r_k}^\zeta + 1} \\ \text{s.t.} \quad & \sum_{k=1}^K \left( P_{r_k}^\zeta + \sum_{l=1, l \neq k}^K \delta_{l,k} \varepsilon_{l,k}^\zeta \right) \cdot \mathcal{I}_{r_k}^\zeta \leq P_{\max}, \end{aligned} \quad (28a)$$

$$\sum_{k=1}^K \mathcal{I}_{r_k}^\zeta = 1, \quad (28b)$$

$$0 \leq P_{r_k}^\zeta \leq E_{\Sigma, r_k}^\zeta \cdot \mathcal{I}_{r_k}^\zeta, \quad \forall k \in \{1, 2, \dots, K\}, \quad (28c)$$

$$\sum_{l=1, l \neq k}^K \varepsilon_{k,l}^\zeta \leq E_{\Sigma, r_k}^\zeta \cdot (1 - \mathcal{I}_{r_k}^\zeta), \quad \forall k \in \{1, 2, \dots, K\}, \quad (28d)$$

$$\mathcal{I}_{r_k}^\zeta \in \{0, 1\}, \quad \forall k \in \{1, 2, \dots, K\}, \quad (28e)$$

$$P_{r_k}^\zeta, \varepsilon_{k,l}^\zeta \geq 0, \quad \forall k, l \in \{1, 2, \dots, K\} \text{ and } k \neq l, \quad (28f)$$

where the fourth constraint ensures that a relay will only transfer energy to another relay if it was not selected; while the rest of the constraints are as before. By the same token of problem **R-OPT-RS** ( $\zeta$ ), problem **OPT-RS-EC** ( $\zeta$ ) can be reformulated as

**R-OPT-RS-EC** ( $\zeta$ ):

$$\max \quad \prod_{i=1}^N \frac{\sum_{k=1}^K \bar{P}_{r_k}^\zeta \xi_{i,k}^\zeta \cdot \mathcal{I}_{r_k}^\zeta}{\sum_{k=1}^K \bar{P}_{r_k}^\zeta \chi_{k,i}^\zeta \cdot \mathcal{I}_{r_k}^\zeta + 1}$$

$$\text{s.t.} \quad \sum_{k=1}^K \bar{P}_{r_k}^\zeta \cdot \mathcal{I}_{r_k}^\zeta \leq P_{\max}, \quad (29a)$$

$$\sum_{k=1}^K \mathcal{I}_{r_k}^\zeta = 1, \quad (29b)$$

$$\bar{P}_{r_k}^\zeta = \bar{E}_{\Sigma, r_k}^\zeta + \sum_{l=1, l \neq k}^K \delta_{l,k} \varepsilon_{l,k}^\zeta, \quad \forall k \in \{1, 2, \dots, K\}, \quad (29c)$$

$$\sum_{l=1, l \neq k}^K \varepsilon_{k,l}^\zeta \leq E_{\Sigma, r_k}^\zeta \cdot (1 - \mathcal{I}_{r_k}^\zeta), \quad \forall k \in \{1, 2, \dots, K\}, \quad (29d)$$

$$\mathcal{I}_{r_k}^\zeta \in \{0, 1\}, \quad \forall k \in \{1, 2, \dots, K\}, \quad (29e)$$

$$\varepsilon_{k,l}^\zeta \geq 0, \quad \forall k, l \in \{1, 2, \dots, K\} \text{ and } k \neq l. \quad (29f)$$

To devise a solution procedure, the optimal SNR-maximizing relay  $R_{\text{OPT-EC}}$  is the one that maximizes the product of LF functions of the objective function when it uses all its harvested energy and “if necessary” cooperation energy from the neighboring relays, according to  $\bar{P}_{r_k}^\zeta$  (see (27)). Since there is a loss associated with wireless energy transfer, then energy cooperation between the relays must be done in accordance with the following proposition.

**Proposition 2:** From an energy perspective, the optimal wireless energy transfer policy to a relay is an ordered sequence, in which the relay with the best wireless transfer efficiency (i.e. closest to that relay) transfers its energy, then the second best/closest relay and so on, until  $P_{\text{max}}$  is reached.

**Proof:** This proposition can be proved by both contradiction and induction. Assume that energy is to be wirelessly transferred to relay  $R_k$ , and let  $\bar{E}_{\Sigma, r_k}^\zeta = E_{\Sigma, r_k}^\zeta < P_{\text{max}}$ . Thus,  $\sum_{l=1, l \neq k}^K \delta_{l,k} \varepsilon_{l,k}^\zeta \leq P_{\text{max}} - E_{\Sigma, r_k}^\zeta$ . Now, let there be three other relays,  $R_l$ ,  $R_q$  and  $R_w$  (for  $l \neq q \neq w \neq k$ ), with relay  $R_l$  ( $R_w$ ) being the closest to (farthest from) relay  $R_k$  (and hence  $0 < \delta_{w,k} < \delta_{q,k} < \delta_{l,k} < 1$ ). Also, assume relays  $R_l$ ,  $R_q$  and  $R_w$  have harvested energies  $E_{\Sigma, r_l}^\zeta$ ,  $E_{\Sigma, r_q}^\zeta$ , and  $E_{\Sigma, r_w}^\zeta$ , respectively. Now, consider only relays  $R_l$  and  $R_q$ , and assume that it is optimal that relay  $R_q$  transfers energy first. Then, it would transfer  $\varepsilon_{q,k}^\zeta = (P_{\text{max}} - E_{\Sigma, r_k}^\zeta) / \delta_{q,k} \leq E_{\Sigma, r_q}^\zeta$ . If  $E_{\Sigma, r_k}^\zeta + \delta_{q,k} \varepsilon_{q,k}^\zeta = P_{\text{max}}$ , then relay  $R_l$  does not need to transfer any energy. However, the energy that would have been transferred by relay  $R_l$  is  $\varepsilon_{l,k}^\zeta = (P_{\text{max}} - E_{\Sigma, r_k}^\zeta) / \delta_{l,k} < E_{\Sigma, r_l}^\zeta$ , and thus  $\varepsilon_{l,k}^\zeta < \varepsilon_{q,k}^\zeta$  since  $\delta_{l,k} > \delta_{q,k}$ , which is a contradiction. Now, consider the case of all three relays,  $R_l$ ,  $R_q$  and  $R_w$ , and assume that relay  $R_l$  is in fact the first to transfer all its energy, such that  $E_{\Sigma, r_k}^\zeta + \delta_{l,k} E_{\Sigma, r_l}^\zeta < P_{\text{max}}$ . Then, if  $R_w$  is the second to transfer its energy, then  $\varepsilon_{w,k}^\zeta = (P_{\text{max}} - E_{\Sigma, r_k}^\zeta - \delta_{l,k} E_{\Sigma, r_l}^\zeta) / \delta_{w,k} \leq E_{\Sigma, r_w}^\zeta$  to reach  $P_{\text{max}}$ . However, the energy that would have been transferred by relay  $R_q$  is  $\varepsilon_{q,k}^\zeta = (P_{\text{max}} - E_{\Sigma, r_k}^\zeta - \delta_{l,k} E_{\Sigma, r_l}^\zeta) / \delta_{q,k} \leq E_{\Sigma, r_q}^\zeta$ , where it is clear that  $\varepsilon_{q,k}^\zeta < \varepsilon_{w,k}^\zeta$  since  $\delta_{q,k} > \delta_{w,k}$ , and hence a contradiction. This process is inductively repeated, ultimately proving the proposition.  $\square$

Hence, problem **R-OPT-RS-EC** ( $\zeta$ ) can be solved using the solution procedure illustrated in Table IV.

## 6. TOTAL RELAY POWER MINIMIZATION

It is intuitive to see that increasing the transmit power of a relay improves the end-to-end SNR; however, it may unnecessarily consume all its harvested energy. In practical wireless networks, some relays may want to store some of the harvested energy for later use in transmitting their own data while satisfying target QoS when relaying for other nodes. Moreover, a relay with low transmit power may not provide a good performance; while a relay with a relatively high transmit power will exhaust its harvested energy too soon. Consequently, it is essential to consider total relay power minimization while meeting target SNR requirements, such that a balance between end-to-end SNR, transmit power consumption, and harvested energy is achieved.

## 6.1. Without Relay Selection

### 6.1.1. Without Energy Cooperation

The optimal total relay power minimization (OPT-TRP-MIN) problem subject to target minimum SNR  $\gamma_{T-MIN}$  per source-destination pair  $S_i - D_i$  is expressed as\*\*

**OPT-TRP-MIN ( $\zeta$ ):**

$$\begin{aligned} \min \quad & \sum_{k=1}^K P_{r_k}^{\zeta} \\ \text{s.t.} \quad & \frac{\sum_{k=1}^K P_{r_k}^{\zeta} \xi_{i,k}^{\zeta}}{\sum_{k=1}^K P_{r_k}^{\zeta} \chi_{r_k,i}^{\zeta} + 1} \geq \gamma_{T-MIN}, \quad \forall i \in \{1, 2, \dots, N\}, \\ & \text{Constraints (13a) and (13b)}. \end{aligned} \quad (30)$$

Although the objective function is linear (and hence convex), this problem is still non-convex, as the first constraint involves an LF function. However, it can be transformed into a linear one as

$$\sum_{k=1}^K P_{r_k}^{\zeta} \xi_{i,k}^{\zeta} \geq \gamma_{T-MIN} \cdot \left( \sum_{k=1}^K P_{r_k}^{\zeta} \chi_{r_k,i}^{\zeta} + 1 \right), \quad \forall i \in \{1, 2, \dots, N\}. \quad (31)$$

Consequently, problem **OPT-TRP-MIN ( $\zeta$ )** is reformulated as

**R-OPT-TRP-MIN ( $\zeta$ ):**

$$\begin{aligned} \min \quad & \sum_{k=1}^K P_{r_k}^{\zeta} \\ \text{s.t.} \quad & \sum_{k=1}^K P_{r_k}^{\zeta} \xi_{i,k}^{\zeta} - \gamma_{T-MIN} \cdot \left( \sum_{k=1}^K P_{r_k}^{\zeta} \chi_{r_k,i}^{\zeta} + 1 \right) \geq 0, \quad \forall i \in \{1, 2, \dots, N\}, \\ & \text{Constraints (13a) and (13b)}, \end{aligned} \quad (32)$$

which is an LP problem that can be solved by any standard optimization software package.

### 6.1.2. With Energy Cooperation

The optimal total relay power minimization with energy cooperation (OPT-TRP-MIN-EC) problem is given by

**OPT-TRP-MIN-EC ( $\zeta$ ):**

$$\begin{aligned} \min \quad & \sum_{k=1}^K \left( P_{r_k}^{\zeta} + \sum_{l=1, l \neq k}^K \delta_{l,k} \varepsilon_{l,k}^{\zeta} \right) \\ \text{s.t.} \quad & \frac{\sum_{k=1}^K \left( P_{r_k}^{\zeta} + \sum_{l=1, l \neq k}^K \delta_{l,k} \varepsilon_{l,k}^{\zeta} \right) \xi_{i,k}^{\zeta}}{\sum_{k=1}^K \left( P_{r_k}^{\zeta} + \sum_{l=1, l \neq k}^K \delta_{l,k} \varepsilon_{l,k}^{\zeta} \right) \chi_{r_k,i}^{\zeta} + 1} \geq \gamma_{T-MIN}, \quad \forall i \in \{1, 2, \dots, N\}, \\ & \text{Constraints (23a) - (23c)}, \end{aligned} \quad (33)$$

which is reformulated as

**R-OPT-TRP-MIN-EC ( $\zeta$ ):**

$$\min \quad \sum_{k=1}^K \left( P_{r_k}^{\zeta} + \sum_{l=1, l \neq k}^K \delta_{l,k} \varepsilon_{l,k}^{\zeta} \right)$$

\*\* An alternative formulation could involve a target SNR  $\gamma_{i,T-MIN}$  per source-destination pair  $S_i - D_i$ . However, this work focuses on the case of common target SNR to all source-destination pairs.

$$\text{s.t. } \sum_{k=1}^K \left( P_{r_k}^\zeta + \sum_{l=1, l \neq k}^K \delta_{l,k} \varepsilon_{l,k}^\zeta \right) \xi_{i,k}^\zeta - \gamma_{\text{T-MIN}} \cdot \left( \sum_{k=1}^K \left( P_{r_k}^\zeta + \sum_{l=1, l \neq k}^K \delta_{l,k} \varepsilon_{l,k}^\zeta \right) \chi_{r_{k,i}}^\zeta + 1 \right) \geq 0, \quad \forall i \in \{1, 2, \dots, N\},$$

Constraints (23a) - (23c), (34)

which is an LP problem that can be easily solved.

## 6.2. With Relay Selection

### 6.2.1. Without Energy Cooperation

The optimal total relay power minimization with relay selection (OPT-TRP-MIN-RS) problem is formulated as

**OPT-TRP-MIN-RS** ( $\zeta$ ):

$$\begin{aligned} \min \quad & \sum_{k=1}^K P_{r_k}^\zeta \cdot \mathcal{I}_{r_k}^\zeta \\ \text{s.t.} \quad & \frac{\sum_{k=1}^K P_{r_k}^\zeta \xi_{i,k}^\zeta \cdot \mathcal{I}_{r_k}^\zeta}{\sum_{k=1}^K P_{r_k}^\zeta \chi_{r_{k,i}}^\zeta \cdot \mathcal{I}_{r_k}^\zeta + 1} \geq \gamma_{\text{T-MIN}}, \quad \forall i \in \{1, 2, \dots, N\}, \\ & \text{Constraints (18a) - (18d),} \end{aligned} \quad (35)$$

which is an MINLP problem. Such problem can be reformulated as

**R-OPT-TRP-MIN-RS** ( $\zeta$ ):

$$\begin{aligned} \min \quad & \sum_{k=1}^K P_{r_k}^\zeta \cdot \mathcal{I}_{r_k}^\zeta \\ \text{s.t.} \quad & \sum_{k=1}^K P_{r_k}^\zeta \xi_{i,k}^\zeta \cdot \mathcal{I}_{r_k}^\zeta - \gamma_{\text{T-MIN}} \cdot \left( \sum_{k=1}^K P_{r_k}^\zeta \chi_{r_{k,i}}^\zeta \cdot \mathcal{I}_{r_k}^\zeta + 1 \right) \geq 0, \quad \forall i \in \{1, 2, \dots, N\}, \\ & \text{Constraints (18a) - (18d).} \end{aligned} \quad (36)$$

To determine the optimal total power minimizing relay, for each relay  $R_k$ , set  $\mathcal{I}_{r_k}^\zeta = 1$  while  $\mathcal{I}_{r_l}^\zeta = 0, \forall l \in \{1, 2, \dots, K\}$  and  $l \neq k$ . Consequently, problem **R-OPT-TRP-MIN-RS** ( $\zeta$ ) becomes an LP problem, which can be easily solved to determine the minimum amount of transmit power of relay  $R_k$  (denoted  $\hat{P}_{r_k}^\zeta$ ) to satisfy the  $\gamma_{\text{T-MIN}}$  constraint. Thus, the relay with the minimum transmit power is the optimal total power minimizing relay  $R_{\text{OPT-TRP-MIN}}$ . The solution procedure for the above optimization problem is summarized in Table V.

### 6.2.2. With Energy Cooperation

The optimal total relay power minimization with relay selection and energy cooperation (OPT-TRP-MIN-RS-EC) problem is given by

**OPT-TRP-MIN-RS-EC** ( $\zeta$ ):

$$\begin{aligned} \min \quad & \sum_{k=1}^K \left( P_{r_k}^\zeta + \sum_{l=1, l \neq k}^K \delta_{l,k} \varepsilon_{l,k}^\zeta \right) \cdot \mathcal{I}_{r_k}^\zeta \\ \text{s.t.} \quad & \frac{\sum_{k=1}^K \left( P_{r_k}^\zeta + \sum_{l=1, l \neq k}^K \delta_{l,k} \varepsilon_{l,k}^\zeta \right) \xi_{i,k}^\zeta \cdot \mathcal{I}_{r_k}^\zeta}{\sum_{k=1}^K \left( P_{r_k}^\zeta + \sum_{l=1, l \neq k}^K \delta_{l,k} \varepsilon_{l,k}^\zeta \right) \chi_{r_{k,i}}^\zeta \cdot \mathcal{I}_{r_k}^\zeta + 1} \geq \gamma_{\text{T-MIN}}, \quad \forall i \in \{1, 2, \dots, N\}, \\ & \text{Constraints (28a) - (28f),} \end{aligned} \quad (37)$$

which can be reformulated as

**R-OPT-TRP-MIN-RS-EC ( $\zeta$ ):**

$$\begin{aligned}
\min \quad & \sum_{k=1}^K \left( P_{r_k}^\zeta + \sum_{l=1, l \neq k}^K \delta_{l,k} \varepsilon_{l,k}^\zeta \right) \cdot \mathcal{I}_{r_k}^\zeta \\
\text{s.t.} \quad & \sum_{k=1}^K \left( P_{r_k}^\zeta + \sum_{l=1, l \neq k}^K \delta_{l,k} \varepsilon_{l,k}^\zeta \right) \xi_{i,k}^\zeta \cdot \mathcal{I}_{r_k}^\zeta - \gamma_{\text{TF-MIN}} \cdot \left( \sum_{k=1}^K \left( P_{r_k}^\zeta + \sum_{l=1, l \neq k}^K \delta_{l,k} \varepsilon_{l,k}^\zeta \right) \chi_{r_k,i}^\zeta \cdot \mathcal{I}_{r_k}^\zeta + 1 \right) \geq 0, \\
& \forall i \in \{1, 2, \dots, N\}, \\
& \text{Constraints (28a) - (28f)}. \tag{38}
\end{aligned}$$

The solution procedure to solving the above problem is similar to that presented in Table V, but with **R-OPT-TRP-MIN-RS** ( $\zeta$ ) replaced by **R-OPT-TRP-MIN-RS-EC** ( $\zeta$ ). Moreover, the relay  $R_k$  with the minimum transmit power and transferred energy (denoted  $\hat{P}_{r_k}^\zeta \triangleq P_{r_k}^\zeta + \sum_{l=1, l \neq k}^K \delta_{l,k} \varepsilon_{l,k}^\zeta$ ) is selected.

**Remark 7:** All proposed solution procedures of the reformulated optimization problems have polynomial-time complexity.

In summary, the formulated power allocation, relay selection and energy cooperation for SNR maximization and total relay power minimization are optimal on a communication frame-by-frame basis. This is due to the fact that the different strategies are performed every communication frame  $\zeta$ , while taking into account the random energy arrivals (modeled as Poisson processes), and instantaneous channel conditions that vary from one from communication frame to another. Moreover, in our network model, a battery-capacity constraint  $B_{\max}$  is assumed at all relays, along with a total power constraint per time-slot  $P_{\max}$  (as per Remarks 1 and 3), which are to be met in every communication frame  $\zeta$ . Furthermore, the different strategies take into account the total harvested energy (as well as any leftover energy from previous transmissions) up to the  $N^{\text{th}}$  time-slot of the current communication frame  $\zeta$  (as per Remarks 4, 5 and 6). In other words, such strategies harmonize the energy consumption for data transmissions with the battery recharge rate by harvesting energy from the random and sporadic RF signals in every communication frame. Lastly, our wireless energy transfer model (for energy cooperation) takes into account the loss associated with it. Hence, our online formulations and solution procedures are optimal, since they take into account randomness in energy arrivals and channel conditions in every communication frame, and thus capture the dynamics of practical energy harvesting networks.

## 7. SIMULATION RESULTS

In this section, the different power allocation and relay selection strategies without and with energy cooperation are evaluated over the network topology shown in Fig. 2. The simulations assume  $P_s = P_{\max} \triangleq P = 100$  mW,  $B_{\max} = 1000$  mW,  $\nu = 2.5$ ,  $\rho = 0.1$ ,  $\eta = 0.025$ , and  $E_{\max, r_k} = 5$  mJ,  $\forall k \in \{1, 2, 3\}$ . Energy arrival rate at each relay  $R_k$  is given by  $\lambda_{r_k} = 4 - k$  arrivals/time-slot for  $k \in \{1, 2, 3\}$ . The simulations are averaged over 5000 independent runs with randomly generated channel coefficients that change from one communication frame to another, over 20 frames for each run.

In Fig. 3, the average end-to-end SNR of the formulated and reformulated SNR-maximizing optimization problems for source-destination pairs  $S_1 - D_1$  and  $S_2 - D_2$  are evaluated<sup>††</sup>. Clearly, the reformulated optimal problems (i.e. R-OPT)—which are solved via the proposed solution procedures—perfectly coincide with those of optimal formulations (i.e. OPT). Moreover, the end-to-end SNR of source-destination pair  $S_1 - D_1$  is greater than that of  $S_2 - D_2$  under the different strategies. This is due to the locations of nodes  $S_1$  and  $D_1$  being relatively closer to the relays than nodes  $S_2$  and  $D_2$

<sup>††</sup> Formulated optimal (OPT) power allocation and relay selection strategies without and with energy cooperation are solved using MIDACO [30], with tolerance set to 0.0001.

(and hence less path-loss). Furthermore, it is evident that the end-to-end SNR values under the power allocation strategies without and with energy cooperation (Figs. 3a and 3c) are greater than those of their relay selection counterparts (Figs. 3b and 3d). This is because under the power allocation strategies, all the harvested energy from the different relays can be *simultaneously* utilized in the multiple-access transmission; while under the relay selection strategies, only the harvested energy at the selected relay and possibly transferred energy are utilized. Lastly, one can see that energy cooperation improves the end-to-end SNR under the different strategies (see Figs. 3a and 3c, and Figs. 3b and 3d).

In Fig. 4, the average available energy at each relay under the different optimal SNR-maximizing strategies is shown. In Fig. 4a, one can see that the available energy at relay  $R_3$  is greater than the other two relays, although it harvests energy at a lower rate. This is due to its location being relatively farther from both source-destination pairs than the other two relays, which in turn implies that its not as effective as the other relays for SNR maximization, and thus its harvested energy is not utilized as much. Moreover, the available energy at relay  $R_1$  is the lowest although it has the highest rate of energy arrivals, which indicates that its harvested energy is effective for SNR maximization and thus often exhausted. Additionally, the available energy for all relays is exhausted at low  $P/N_0$  values, but starts to increase at relatively high values of  $P/N_0$ . This is because the resulting end-to-end SNR starts to saturate for high enough  $P/N_0$  values (see Fig. 3a) and thus, some of the available energy becomes redundant. Fig. 4b shows that relay  $R_2$  has the lowest available energy although it has the second highest energy arrivals rates, but due to its location, it is more effective for SNR maximization when selected (as will be illustrated in Fig. 5). Additionally, relay selection without energy cooperation (see Fig. 4b) results in higher energy availability than power allocation without energy cooperation (see Fig. 4a), as only one relay is active in any cooperation phase, and thus relays get to store more energy on average. Fig. 4c shows that the available energy at relay  $R_2$  decreases with the increase in  $P/N_0$ , while that of relays  $R_1$  and  $R_3$  increases. This implies that it is more effective for SNR maximization for relay  $R_2$  to utilize its energy for relaying and possibly energy cooperation with relays  $R_1$  and  $R_3$ . One can also see that the available energy at the relays with energy cooperation (see Fig. 4c) is higher than that without it (see Fig. 4a). Thus, joint power allocation and energy cooperation retains more energy at the relays, despite the loss associated with wireless energy transfer. Finally, Fig. 4d shows that all the energy is exhausted for joint relay selection and energy cooperation, as all the harvested energy at a relay is wirelessly transferred to the selected relay, which uses it along with its harvested energy for cooperative relaying. This has happened in our simulations because the total harvested and transferred energy at the optimal relay is always less than  $P_{\max}$ . If this had not been the case, then some relays would have retained some of their harvested energy.

Figs. 5a and 5b illustrate the percentage of SNR-maximizing relay selection without and with energy cooperation at  $P/N_0 = 40$  dB, respectively. In Fig. 5a, one can see that relays  $R_1$  and  $R_2$  are selected %52 and %43 of the time, respectively, while relay  $R_3$  is selected only %5 of the time. However, in Fig. 5b, it can be seen that relay  $R_2$  is selected about %88 of the time, despite its energy arrival rate not being the greatest. This is due to relay  $R_2$ 's location being relatively closer to both source-destination pairs, which makes selecting this relay more effective in improving the end-to-end SNR when combined with energy cooperation.

In Fig. 6, the average total relay power is compared under the different optimal total relay power-minimizing strategies when  $P/N_0 = 40$  dB and  $\gamma_{T-\text{MIN}} = 3$  dB. It is clear that the optimal problem formulations coincide with the reformulated solutions. In addition, one can see that power allocation without energy cooperation (Fig. 6a) requires more relay power than that with energy cooperation (Fig. 6c). This is because with energy cooperation, it possible that a relay with bad channel conditions but high enough harvested energy, to share its energy with other relays that have better channel conditions, ultimately satisfying the target end-to-end SNR  $\gamma_{T-\text{MIN}}$  with less total relay power. On the other hand, relay selection with energy cooperation (Fig. 6d) yields lower total relay power than that without energy cooperation (Fig. 6b). This is explained by noting that a relay with good channel conditions but low harvested energy may still be selected, as

energy can be transferred to it from the other relays to satisfy  $\gamma_{T-MIN}$  with the minimum amount of relay power. Overall, the power allocation strategies require less total relay power than their relay selection counterparts, as all the relays can participate in cooperative transmission, such that  $\gamma_{T-MIN}$  is satisfied<sup>‡‡</sup>

Fig. 7 illustrates the average end-to-end SNR of source-destination pairs  $S_1 - D_1$  and  $S_2 - D_2$  under total relay power minimization. It is evident that both source-destination pairs satisfy the required  $\gamma_{T-MIN}$ , with source-destination pair  $S_1 - D_1$  achieving higher end-to-end SNR than  $S_2 - D_2$  under the different strategies, in agreement with the results presented in Fig. 3. Moreover, for the  $S_1 - D_1$  pair, the **TRP-MIN-ORS** strategy yields higher end-to-end SNR than **TRP-MIN-ORS-EC**, as it requires greater total relay transmit power (see Figs. 6b and 6d). In addition, the end-to-end SNR of **TRP-MIN-OPA-EC** strategy is lower than that of **TRP-MIN-OPA**. This is attributed to the efficient sharing of harvested energy between the relays such that the total relay transmit power is minimized (in agreement with Figs. 6a and 6c). Furthermore, due the locations of nodes  $S_2$  and  $D_2$  being relatively farther from the relays than nodes  $S_1$  and  $D_1$ , then intuitively, it is more important to select a relay that satisfies  $\gamma_{T-MIN}$  for  $S_2 - D_2$ , which will most likely satisfy it for the  $S_1 - D_1$  pair, and yield—on average—higher end-to-end SNR for it. Overall, the power allocation strategies yield lower end-to-end SNR than the relay selection strategies, as they require less total relay transmit power to satisfy  $\gamma_{T-MIN}$ .

In Figs. 8a and 8b, the average relay selection probabilities for total relay power minimization without and with energy cooperation are illustrated. In both figures, it is clear that relay  $R_2$  is selected much more often than the other two relays, which is due to its location being relatively closer to both source-destination pairs than the other relays. Moreover, the probability of selecting relay  $R_2$  is greater with energy cooperation, as noted earlier. On the other hand, relay  $R_3$  is selected more often than relay  $R_1$ . This is interpreted by noting that source-destination pair  $S_2 - D_2$  is relatively farther from relay  $R_1$  than source-destination pair  $S_1 - D_1$ . Thus, to minimize the total relay transmit power, both source-destination pairs are better served by relays  $R_2$  and  $R_3$  to satisfy  $\gamma_{T-MIN}$ .

Fig. 9 compares the end-to-end SNR of source-destination pairs  $S_1 - D_1$  and  $S_2 - D_2$ —when  $P/N_0 = 40$  dB—of the different optimal SNR-maximizing power allocation and relay selection (without and with energy cooperation) strategies with equal power allocation (EPA), and random relay selection (RRS). Under the EPA strategy, the transmit powers of the relays are equal; while under RRS strategy, a single relay is randomly selected for cooperative transmission. In Figs. 9a and 9b, one can see that the EPA and RRS strategies yield significantly less end-to-end SNR under both source-destination pairs than the different optimal power allocation and relay selection strategies. Additionally, the obtained end-to-end SNR under RRS is worse than that of the EPA, in agreement with previous results that demonstrated that the power allocation strategies are superior to their relay selection counterparts.

In Fig. 10a, the average total relay power of the different optimal total relay power-minimizing strategies—when  $P/N_0 = 40$  dB and  $\gamma_{T-MIN} = 3$  dB—are compared with those of the **TRP-MIN-EPA** and **TRP-MIN-RRS**. It is evident that the **TRP-MIN-RRS** strategy is the worst in terms of the total relay power, which is followed by the **TRP-MIN-EPA** strategy. Figs. 10b and 10c illustrate the end-to-end SNR of the source-destination pairs  $S_1 - D_1$  and  $S_2 - D_2$ , respectively, where it can be seen that although the **TRP-MIN-EPA** and **TRP-MIN-RRS** strategies satisfy  $\gamma_{T-MIN}$  under both source-destination pairs, the end-to-end SNR of the **TRP-MIN-RRS** strategy is higher than all the other strategies for both source-destination pairs. This is in agreement with the observation made in Fig. 10a that it requires the greatest total relay power. Therefore, the optimal strategies require less total relay power in order to closely satisfy  $\gamma_{T-MIN} = 3$  dB than the **TRP-MIN-EPA** and **TRP-MIN-RRS** strategies.

<sup>‡‡</sup>Since all the optimal and reformulated optimal SNR-maximizing and total relay power-minimizing strategies have been shown to yield identical results in terms of the end-to-end SNR and total relay power, respectively, then the SNR-maximizing **OPT-PA** and **R-OPT-PA** (**OPT-PA-EC** and **R-OPT-PA-EC**) strategies are collectively referred to as **OPA** (**OPA-EC**). Similarly, their relay selection counterparts are denoted **ORS** and **ORS-EC**. As for the total relay power-minimizing strategies, the **OPT-TRP-MIN** and **R-OPT-TRP-MIN** (**OPT-TRP-MIN-EC** and **R-OPT-TRP-MIN-EC**) strategies are collectively referred to as **TRP-MIN-OPA** (**TRP-MIN-OPA-EC**). Similarly, the relay selection strategies are collectively denoted **TRP-MIN-ORS** and **TRP-MIN-ORS-EC**.



## 8. CONCLUSIONS

In this paper, power allocation and relay selection strategies for end-to-end SNR-maximization without and with energy cooperation are studied. In addition, total relay power minimization subject to target end-to-end SNR is considered to maintain QoS. Particularly, the different strategies are formulated as optimization problems, which are non-convex. However, by applying intelligent transformations, such problems have been converted into convex ones. After that, polynomial-time solution procedures have been proposed and verified to coincide with the optimal non-convex formulations. Additionally, it has been shown that power allocation strategies achieve higher end-to-end SNR than the relay selection strategies. Furthermore, energy cooperation has been shown to be effective in improving end-to-end SNR and minimizing total relay transmit power when augmented with the power allocation and relay selection strategies.

## REFERENCES

1. Xiao L., Wang P., Niyato D., Han Z. Wireless networks with RF energy harvesting: A contemporary survey. *IEEE Communications Surveys and Tutorials* 2015; 17 (2): 757–789.
2. Ulukus S., Yener A., Erkip E., Simeone O., Zorzi M., Grover P., Huang K. Energy harvesting wireless communications: a review of recent advances. *IEEE Journal on Selected Areas in Communications* 2015; 33 (3): 360–381.
3. Zanella A., Bazzi A., Masini B., Pasolini G. Optimal transmission policies for energy harvesting nodes with partial information of energy arrivals. *Proc. of IEEE 19th International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC)* 2013: 954–959.
4. Gurakan B., Ozel O., Yang J., Ulukus S. Energy cooperation in energy harvesting wireless communications. *Proc. of IEEE International Symposium of Information Theory (ISIT)* 2012: 965–969.
5. Ahmed I., Ikhlef A., Schober R., Mallik R. Power allocation in energy harvesting relay systems. *Proc. of IEEE Vehicular Technology Conference (VTC)* 2012: 1–5.
6. Ahmed I., Ikhlef A., Schober R., Mallik R. Joint power allocation and relay selection in energy harvesting AF relay systems. *IEEE Wireless Communications Letters* 2013; 2 (2): 239–242.
7. Luo Y., Zhang J., Letaief K. Achieving energy diversity with multiple energy harvesting relays. *Proc. of IEEE Wireless Communications and Signal Processing (WCSP)* 2014: 1–6.
8. Luo Y., Zhang J., Letaief K. Relay selection for energy harvesting cooperative communication systems. *Proc. of IEEE Global Communications Conference (GLOBECOM)* 2013: 2514–2519.
9. Ozel O., Tutuncuoglu K., Yang J., Ulukus S., Yener A. Transmission with energy harvesting nodes in fading wireless channels: Optimal policies. *IEEE Journal on Selected Areas in Communications* 2011; 29 (8), 1732–1743.
10. Orhan O., Erkip E. Throughput maximization for energy harvesting two-hop networks. *Proc. of IEEE International Symposium on Information (ISIT)* 2013; 1596–1600.
11. Ho C., and Zhang R. Optimal energy allocation for wireless communications with energy harvesting constraints. *IEEE Transactions on Signal Processing* 2012; 60 (9), 4808–4814.
12. Antepi M., Uysal-Biyikoglu E., Erkal H. Optimal packet scheduling on an energy harvesting broadcast link. *IEEE Journal on Selected Areas in Communications* 2011; 29 (8), 1721–1731.
13. Yang J., Ozel O., Ulukus S. Broadcasting with an energy harvesting rechargeable transmitter. *IEEE Transactions on Wireless Communications* 2012; 11 (2): 571–583.
14. Ozel O., Yang J., Ulukus S. Optimal broadcast scheduling for an energy harvesting rechargeable transmitter with a finite capacity battery. *IEEE Transactions on Wireless Communications* 2012; 11 (6), 2193–2203.

15. Yang J., Ulukus S. Optimal packet scheduling in a multiple access channel with energy harvesting transmitters". *Journal of Communications and Networks* 2012; 14 (2), 140–150.
16. Tutuncuoglu K., Yener A. Cooperative energy harvesting communications with relaying and energy sharing. *Proc. of IEEE Information Theory Workshop (ITW)* 2013: 1–5.
17. Medepally B., Mehta N. Voluntary energy harvesting relays and selection in cooperative wireless networks. *IEEE Transactions on Wireless Communications* 2010; 9 (11): 3543–3553.
18. Bonami P., Kilinc M., Linderoth J. Algorithms and software for convex mixed integer nonlinear programs. *The IMA Volumes in Mathematics and its Applications* 2012; 154: 1–39.
19. Mitran P. On optimal online policies in energy harvesting systems for compound poisson energy arrivals. *Proc. of IEEE International Symposium of Information Theory (ISIT)* 2012: 960–964.
20. Baidas M., MacKenzie A. Many-to-many space-time network coding for amplify-and-forward cooperative networks: Node selection and performance analysis. *EURASIP Journal on Wireless Communications and Networking* 2014. DOI: 10.1186/1687-1499-2014-48.
21. Boyd S., Vandenberghe L. *Convex Optimization*. Cambridge University Press, 2004.
22. Schaible S., Shi J. Recent developments in fractional programming: Single ratio and max-min case. *Proc. of International Conference on Nonlinear Analysis and Convex Analysis* 2003: 493–506.
23. Freund R., Jarre F. Solving the sum-of-ratios problem by an interior point method. *Journal of Global Optimization* 2001; 19: 83–102.
24. Hirche J. Optimizing of sums and products of linear fractional functions under linear constraints. *Optimization: A Journal of Mathematical Programming and Operations Research* 1996; 38 (1): 39–48.
25. Avriel M., Diewart W., Schaible S., Zang I. *Generalized Convexity*. Plenum Press, N. Y., 1988.
26. Guzel N. A proposal to the solution of multiobjective linear fractional programming problem. *Abstract and Applied Analysis* 2013; 11. DOI: 10.1155/2013/435030.
27. Kannan R., Monma C. On the computational complexity of integer programming problems. *Optimization and Operations Research: Lecture Notes in Economics and Mathematical Systems* 1978; 157: 161–172.
28. Matsui T., Saruwatari Y., Shigeno M. An analysis of Dinkelback's algorithms for 0-1 fractional programming problems. Technical Reports METR92-14, Department of Mathematical Engineering and Information Physics, University of Tokyo, 1992.
29. Prasad R., Devasenapath S., Rao V., Vazifehdan J. Reincarnation in the ambience: Devices and networks with energy harvesting. *IEEE Communications Surveys and Tutorials* 2013; 16 (1): 195–213.
30. Schlueter M. MIDACO software performance on interplanetary trajectory benchmarks. *Advances in Space Research* 2014; 54 (4): 744–754.

**Table I.** Solution Procedure for Optimal SNR-Maximizing Power Allocation

- 
- Step 1:** Determine  $Z_i^{\zeta}, \forall i \in \{1, 2, \dots, N\}$ .  
**Step 2:** Solve problem **R-OPT-PA** ( $\zeta$ ) to obtain  $\mathbf{p}_r^{\zeta}$ .
- 

**Table II.** Solution Procedure for Optimal SNR-Maximizing Relay Selection

- 
- Step 1:** For each  $k \in \{1, 2, \dots, K\}$ ,  
**Step 1.1:** Set  $\mathcal{I}_{r_k}^{\zeta} = 1$  and  $\mathcal{I}_{r_l}^{\zeta} = 0, \forall l \in \{1, 2, \dots, K\}$  and  $l \neq k$ .  
**Step 1.2:** Evaluate  $\Pi_{RS,k} = \prod_{i=1}^N \frac{\sum_{k=1}^K \bar{\epsilon}_{i,k}^{\zeta} \cdot \mathcal{I}_{r_k}^{\zeta}}{\sum_{k=1}^K \bar{x}_{r_k,i}^{\zeta} \cdot \mathcal{I}_{r_k}^{\zeta} + 1}$ .  
**Step 2:** Determine  $R_{OPT} = \arg \max_{k \in \{1, 2, \dots, K\}} \Pi_{RS,k}$ , and  $\Pi_{OPT-RS} = \max_{k \in \{1, 2, \dots, K\}} \Pi_{RS,k}$ .
- 

**Table III.** Solution Procedure for Optimal SNR-Maximizing Joint Power Allocation and Energy Cooperation

- 
- Step 1:** Determine  $\bar{Z}_i^{\zeta}, \forall i \in \{1, 2, \dots, N\}$ .  
**Step 2:** Solve problem **R-OPT-PA-EC** ( $\zeta$ ) to obtain  $P_{r_k}^{\zeta}, \epsilon_{k,l}^{\zeta}, \forall k, l \in \{1, 2, \dots, K\}$  and  $k \neq l$ .
- 

**Table IV.** Solution Procedure for Optimal SNR-Maximizing Joint Relay Selection and Energy Cooperation

- 
- Step 1:** For each  $k \in \{1, 2, \dots, K\}$ ,  
**Step 1.1:** Determine  $\bar{P}_{r_k}^{\zeta}$ , and set  $\mathcal{I}_{r_k}^{\zeta} = 1$  and  $\mathcal{I}_{r_l}^{\zeta} = 0, \forall l \in \{1, 2, \dots, K\}$  and  $l \neq k$ .  
**Step 1.2:** Evaluate  $\Pi_{RS-EC,k} = \prod_{i=1}^N \frac{\sum_{k=1}^K \bar{P}_{r_k}^{\zeta} \epsilon_{i,k}^{\zeta} \cdot \mathcal{I}_{r_k}^{\zeta}}{\sum_{k=1}^K \bar{P}_{r_k}^{\zeta} \times_{k,i}^{\zeta} \cdot \mathcal{I}_{r_k}^{\zeta} + 1}$ .  
**Step 2:** Determine  $R_{OPT-EC} = \arg \max_{k \in \{1, 2, \dots, K\}} \Pi_{RS-EC,k}$ , and  $\Pi_{OPT-RS-EC} = \max_{k \in \{1, 2, \dots, K\}} \Pi_{RS-EC,k}$ .
- 

**Table V.** Solution Procedure For Optimal Total Power Minimization with Relay Selection

- 
- Step 1:** For each  $k \in \{1, 2, \dots, K\}$ ,  
**Step 1.1:** Set  $\mathcal{I}_{r_k}^{\zeta} = 1$  and  $\mathcal{I}_{r_l}^{\zeta} = 0, \forall l \in \{1, 2, \dots, K\}$  and  $l \neq k$ .  
**Step 1.2:** Solve problem **R-OPT-TRP-MIN-RS** ( $\zeta$ ) to determine  $\hat{P}_{r_k}^{\zeta}$ .  
**Step 2:** Determine  $R_{OPT-TRP-MIN} = \arg \min_{k \in \{1, 2, \dots, K\}} \hat{P}_{r_k}^{\zeta}$ .
-

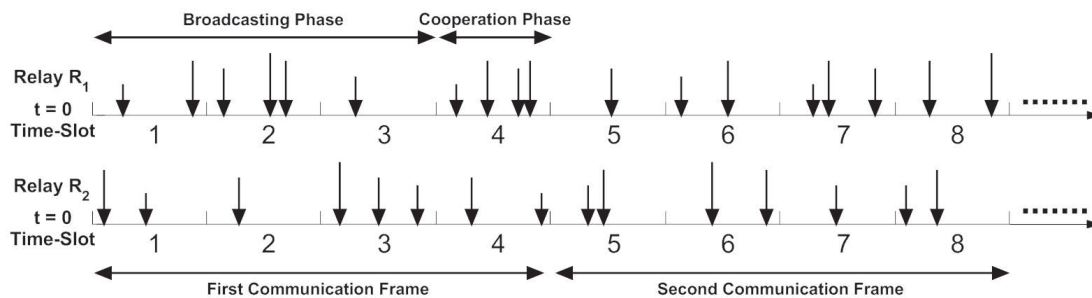


Figure 1. Energy Arrivals of a Network with  $N = 3$  and  $K = 2$  Source and Relay Nodes, Respectively

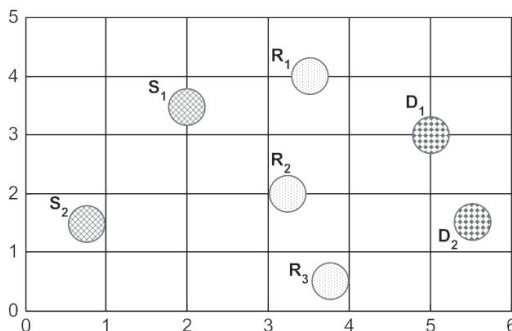


Figure 2. Network Topology

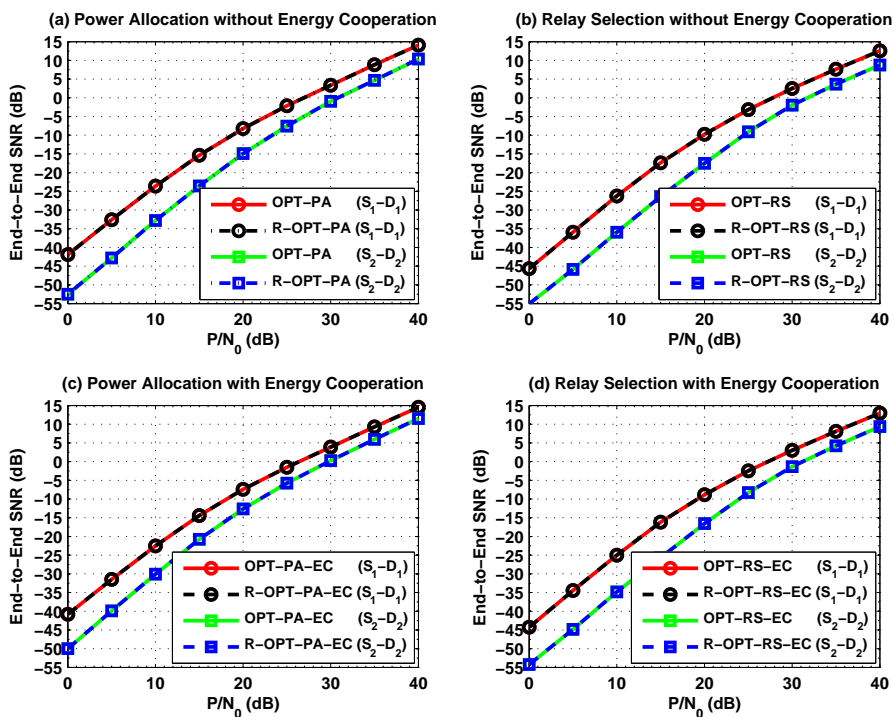


Figure 3. SNR-Maximization: Average End-to-End SNR (dB) of Each Source-Destination Pair

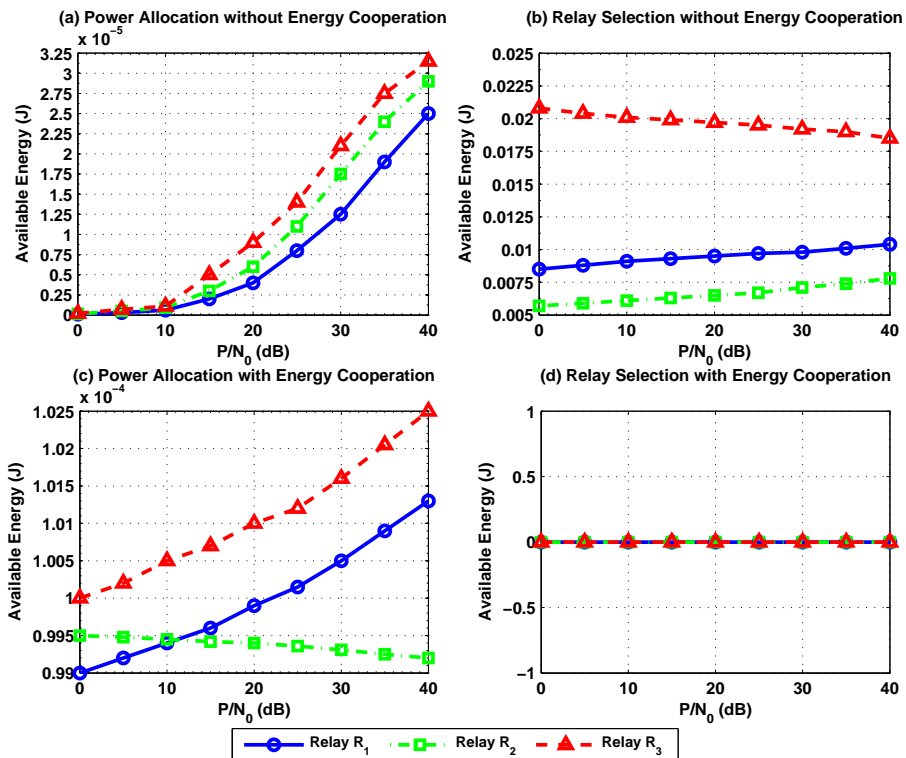


Figure 4. SNR-Maximization: Average Available Energy (J) at Each Relay Node

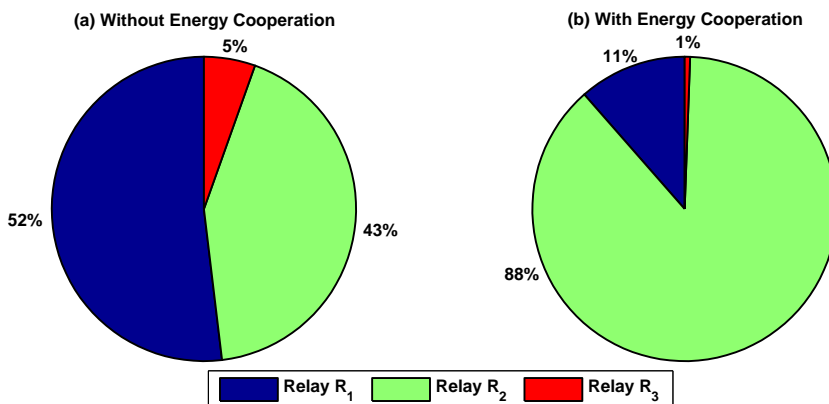


Figure 5. SNR-Maximization: Average Relay Selection (a) Without Energy Cooperation, and (b) With Energy Cooperation -  $P/N_0 = 40$  dB

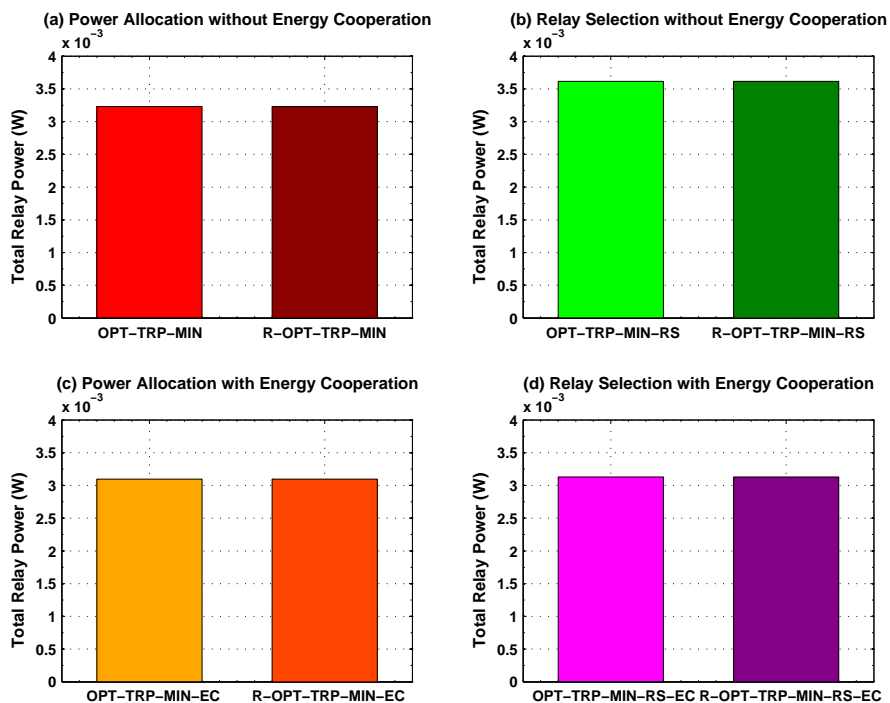


Figure 6. Total Relay Power Minimization: Average Total Relay Power (W) -  $P/N_0 = 40$  dB and  $\gamma_{T-MIN} = 3$  dB

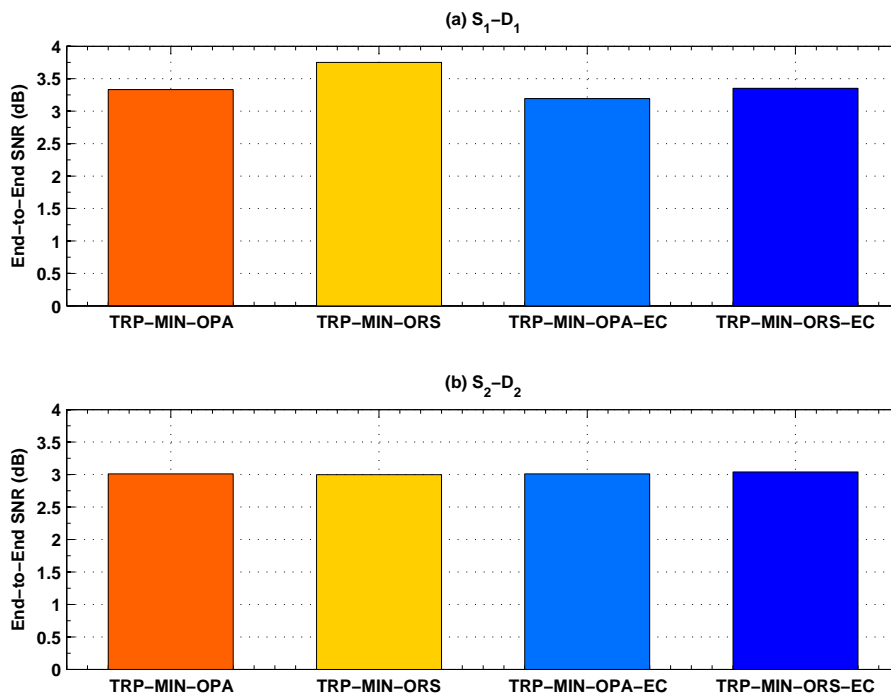
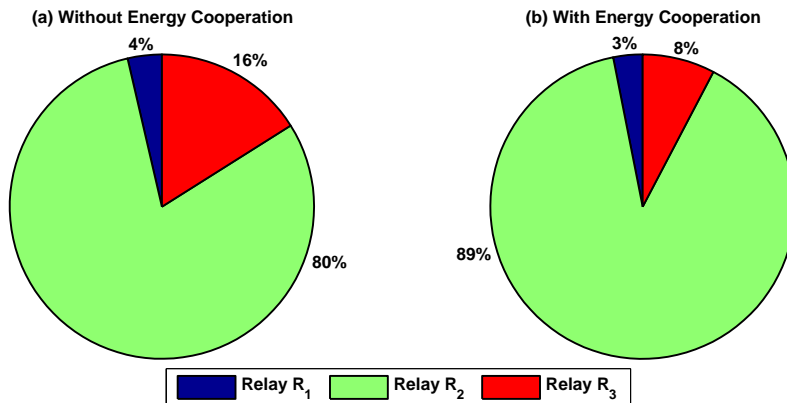
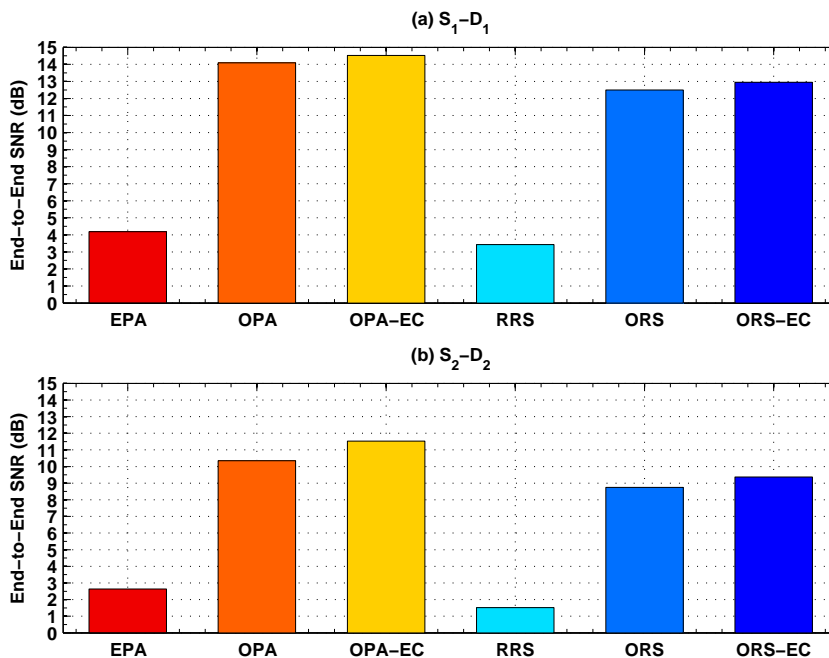


Figure 7. Total Relay Power Minimization: Average End-to-End SNR (dB) of Each Source-Destination Pair -  $P/N_0 = 40$  dB and  $\gamma_{T-MIN} = 3$  dB



**Figure 8.** Total Relay Power Minimization: Average Relay Selection (a) Without Energy Cooperation, and (b) With Energy Cooperation -  $P/N_0 = 40$  dB and  $\gamma_{T-MIN} = 3$  dB



**Figure 9.** SNR-Maximization: Comparison of End-to-End SNR of Source-Destination Pair: (a)  $S_1 - D_1$ , and (b)  $S_2 - D_2$  -  $P/N_0 = 40$  dB

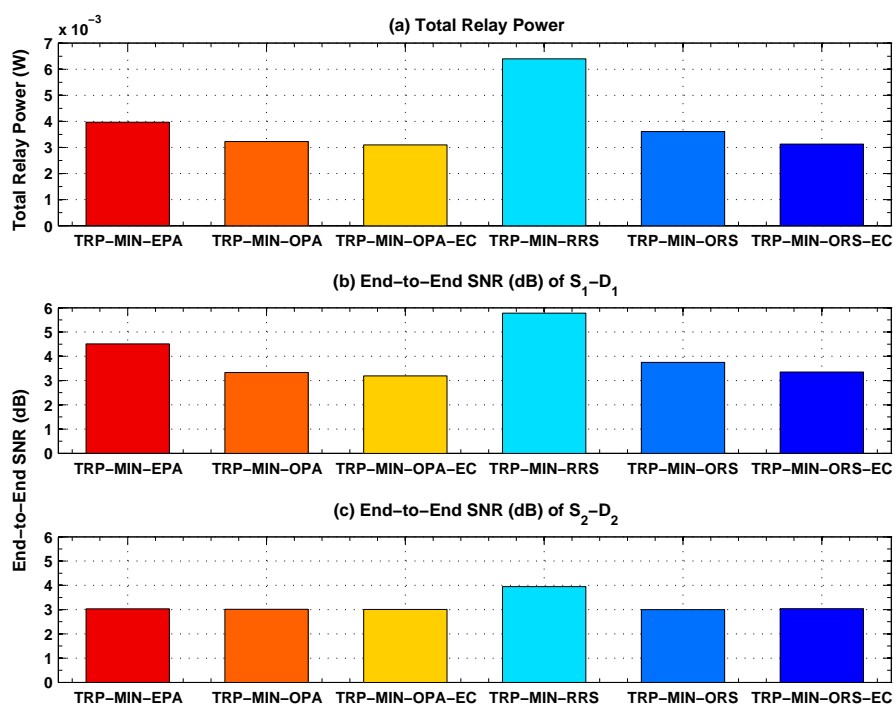


Figure 10. Total Relay Power Minimization: Comparison of (a) Average Total Relay Power, and End-to-End SNR of Source-Destination Pair: (b)  $S_1 - D_1$ , and (c)  $S_2 - D_2$  -  $P/N_0 = 40$  dB and  $\gamma_{T-MIN} = 3$  dB