STRUCTURAL OPTIMISATION OF A COMPOSITE AIRCRAFT FRAME FOR A CHARACTERISTIC RESPONSE CURVE

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Abstract
In this paper an heuristic optimisation technique for a characteristic response curve of CFRP z-frames used in aerospace applications is investigated. Therein, the focus is to identify a dynamic response being most suitable to map onto a specified reference curve. For that purpose a parametric finite element model was established. Discrete laminate stacks and continuous geometry parameters were used as design variables for structural optimisation. In order to solve the nonlinear unconstrained optimisation task, a mixed-integer distributed ant colony algorithm was selected and applied. Within the solution space, a structural layout of the z-frame was identified from a set of pareto optimal solutions being the best solution with respect to the optimisation objectives. Ultimately, the best solution of the response curve approach is compared to the solution of a response value approach.

1. Introduction
Projects on aircraft fuselage development pursue the application of carbon fibre reinforced plastics (CFRP) to achieve enhanced structural efficiency. Once loaded beyond their elastic thresholds, CFRPs mostly exhibit brittle fracture behaviour, which further limits the energy absorption capability by large deformation. The integration of energy absorption functionality into structural elements seems advantageous with respect to the overall fuselage mass balance. With respect to CFRP constitutive properties, energy absorption principles such as fibre breakage, matrix cracking and friction [1] are adaptable. [2] Large deformation of structural elements in the sub-floor area of CFRP fuselage are in focus to exploit energy absorption capabilities. The value of large frame deformation for the overall deformation kinematics was examined by Heimbs et al. [3] and Weiß et al. [2]. Since the frame is the circumferential stiffener of the CFRP fuselage design, its deformation characteristics are provoking the overall deformation kinematics [4]. For favourable manipulation of the overall deformation kinematics of the sub-floor area under crash load conditions, a certain characteristic response of the frame is desired. The desired characteristic response curve was obtained from a preparative design investigation which simultaneously considered static and crash loads in a multi-objective approach as outlined in Weiß et al. [2]. Therefore, retrieving a detailed design in accordance with the characteristic response curve is subject to the present structural optimisation of an composite aircraft frame, see section 2.2 for more details.

Consequently, an optimisation technique for the minimisation of response curve deviation is proposed. The approach is inspired by Pedersen [5], who developed crashworthy designs based on desired acceleration paths. The proposed method relies on the simultaneous consideration of mixed variables for discrete
laminate stacks (integer values) and continuous geometry parameters (floating-point values). Due to the
highly nonlinear nature of the model and the presence of non-relaxable integer variables, an evolutionary
algorithm is suggested to solve the optimisation task. Kicinger et al. [6] examined evolutionary com-
putation in application to structural design with respect to the implications of integer type variables on the
optimisation task. A more general perspective on multi-objective evolutionary algorithms is provided by
Zhou et al. [7], who examined evolutionary algorithms, as for example ant colony optimisation, and their
typical fields of application. The mixed-integer distributed ant colony algorithm (MIDACO) developed
by Schlueter [8] was finally selected, as it is well applicable to mixed-integer nonlinear problems and
benefits from its massive parallelisation capability [9].

Figure 1. Overview on design and modelling details of the composite aircraft z-frame. Section 1 refers
to the outer flange, while section 3 indicates the inner flange. The figures are reproduced from previous
publications by the first author [2, 4].

2. Methodology

2.1. Numerical Calculation

For the sake of conciseness, the following description and figures 1(a)-1(c) on the numerical calcula-
tion are reproduced as excerpts from earlier publications by the first author [4, 10]. More details on
the numerical modelling and parameter settings as well as their selection criteria are available therein.
Essentially, non-linear explicit finite element analyses were performed using LS-Dyna as a solver. The
z-section frame was meshed using shell elements and material model MAT54 with properties provided
by Feraboli et al. [11]. The majority of the remaining control options were set to default, following
the suggestions of the LS-Dyna manual [12]. As outlined in Weiß et al. [2], the z-frame was extracted
from a stiffened panel fuselage design studied previously, see figure 1(a). It was considered feasible to
investigate the frame individually. Since the frame was not directly bonded to the skin or stringers, a
generous clearance was present between those parts. Instead, the frame was connected to the fuselage
skin via clip elements, which constituted an elastic restraint allowing large frame deformations prior to
part contact in a loaded state. Admittedly, investigating the individual frame by itself is a strong simpli-
fication. The obtained parameter set should be subject to verification within the more complex fuselage
assembly. However, this paper intends to investigate an optimisation technique based on a conveniently
simplified model. [2]

Figure 1(c) depicts the cross section view of the z-frame under investigation. Therein, the major dimen-
sions as well as the three constituting segments are shown. The overall height $h$, the flange widths $w_{1,2}$
and the segment thicknesses $t_{1,2,3}$ as functions of the segment laminate stacks $s_{1,2,3}$ are provided. The laminate stacks and the flange widths will be used as variables within the optimisation outlined below. A side view of the curved frame showing the boundary conditions, the angular velocity $\varphi$, the length $l$ as well as the initial curvature $\kappa$ is depicted in figure 1(b). For all calculations, the initial curvature was set to $\kappa = 0.35 \text{m}^{-1}$, which corresponds to A350 or 787 aircraft fuselage configurations. The boundary conditions applied simulate pure bending, which in reality is achieved by a four point bending test [3]. Considering the aircraft fuselage as well as its various load cases, pure bending is hardly present as an isolated loading state. Usually, combined loads of bending and compression or tension act on the frame. For the sake of simplicity, pure bending was applied for all calculations featured in this paper. A linear angular rotation versus time relationship was defined based on data provided by Heimbs et al. [3]. The curve was defined up to $9^\circ$ rotation at 6 ms time. [4]

The computational technique developed in [4] utilises Python [13] as a framework for automated analysis and data processing. The framework consists of the three parts: pre-processing, solving (LS-Dyna), and post-processing, which allows automated structural analysis with different input parameters. With respect to the optimisation task, the dynamic response, the related deformation work, and the frame mass were extracted in the post-processing part. Since explicit analyses with a time stepping scheme were executed, the post-processing involved the identification of a unique critical time step. Therein, the critical time step marked the onset of structural failure. Figure 2 shows a typical structural response of the investigated z-frames. The bending moment and the corresponding deformation energy are plotted versus the rotation. While the bending moment was obtained at the frame centre ($l/2$), the rotation was recorded at the free ends. The black vertical line indicates the detection of failure by the material model. The dotted part of the curves indicate post failure or nonelastic response. [4]

2.2. Structural Optimisation

The purpose of this methodology is to identify a z-frame design most suitable to map onto the reference characteristic response curve. In other words, the design variables of the preliminary design are

![Graph showing deformation work and moment versus rotation](image-url)
subject to modification in order to obtain a structural response curve being equal to the reference curve. The reference curve selected for this investigation is based on a preparative study which simultaneously considered static and crash loads in a multi-objective approach as outlined in Weiß et al. [2]. However, the selection of the reference curve may as well be based on experience or experimental studies such as Waimer [14]. The equality between the structural response curve and the reference curve is assessed by the deviation of the two curves including the curve shape as well as the deformation work. The optimisation is classified as an unconstrained nonlinear mixed-integer optimisation problem. This classification is based on the discontinuity of the structural response of the shock loaded curved z-frame and on the design variables being a mix of continuous and integer values. The algorithm parameters were set to default values while job parallelisation was used.

2.2.1. Optimisation Algorithm

Since the optimisation task is a mixed-integer nonlinear problem, the MIDACO Version 5.0 algorithm [9] was employed. MIDACO is a mixed-integer distributed ant colony optimisation algorithm, which is a global optimisation algorithm based on the stochastic approach of the ant colony metaheuristic. It is able to handle discontinuous problem functions as well as discrete variables without relaxation, see Schlueter et al. [9].

2.2.2. Optimisation Model

The optimisation model is unconstrained, features two continuous and three integer design variables and consists of three objectives, see (1). The variables are provided to the optimisation as a vector of design variables $\mathbf{x}$, see equation (2). First, objective 1 represents the equality between the structural response curve of the current design and the desired reference curve. The measure of equality is the sum of the least squares deviation [13] of the function values (bending moment $M$) between the current $M_{\text{cur}}(\mathbf{x})$ and the reference response curve $M_{\text{ref}}$. Both curves feature a data resolution of 1000 equidistant points along the abscissa, which specifies the control variable $p = 1 \ldots 1000$. $M_{\text{cur}}(\mathbf{x})$ is a function of the vector of design variables $\mathbf{x}$.

Operating with only one objective may not necessarily yield the desired dynamic response. Although a design may be found with ideal equality $f_1(\mathbf{x}) = 0$, it may not achieve the desired deformation work. The option of introducing a second objective was pursued instead of establishing a constraint. The benefit of a second objective over a constraint is: instead of excluding solutions from the set of feasible solutions all solutions remain feasible and constitute a set of compromise solutions. The second objective represents the capability of performing deformation work. The desired deformation work $W_{\text{ref}}$ is related to the current deformation work $W_{\text{cur}}(\mathbf{x})$. Both values of deformation work are computed by numerical integration of the respective response curves. Therein, the specified maximal rotation of the reference curve is used as the upper boundary in the integration. Since the obtained current response curve is available for larger values of rotation then specified by the reference curve, the upper boundary is an important value to consider.

Besides the desire to achieve a certain shape of the response curve and a certain amount of deformation work, the z-frame design is supposed to be a lightweight design as well. Hence, a third objective is introduced to account for that. Objective 3 represents the mass $m(\mathbf{x})$ of the z-frame.

The three objectives, which are combined to the vector $\mathbf{F}(\mathbf{x})$, are subject to minimisation, see equation (2). For a multi-objective optimisation, MIDACO requires the specification of a superior objective function $f_0(\mathbf{x})$ that assists to identify a best solution from the set of pareto optimal solutions, see equation (3). The superior objective is the weighted sum of the three objectives outlined above. The weights $\nu_{1,2,3}$ are not used to identify the pareto front itself, but the best solution based on a specific weighting proposed by the
Table 1. Summary on the design variables, weighting factors, result values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Type</th>
<th>Lower Boundary</th>
<th>Upper Boundary</th>
<th>Best Solution $f_0(x)$</th>
<th>Best Solution $f_0(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$ (mm)</td>
<td>constant</td>
<td>80.00</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$l$ (mm)</td>
<td>constant</td>
<td>1000.00</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$w_1$ (mm)</td>
<td>continuous</td>
<td>14.00</td>
<td>28.00</td>
<td>28.00</td>
<td>14.01</td>
</tr>
<tr>
<td>$w_2$ (mm)</td>
<td>continuous</td>
<td>14.00</td>
<td>28.00</td>
<td>26.38</td>
<td>28.00</td>
</tr>
<tr>
<td>$s_1$</td>
<td>integer</td>
<td>0</td>
<td>26</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>$s_2$</td>
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<td>0</td>
<td>26</td>
<td>23</td>
<td>26</td>
</tr>
<tr>
<td>$s_3$</td>
<td>integer</td>
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<td>26</td>
<td>18</td>
<td>7</td>
</tr>
<tr>
<td>$\nu_1$</td>
<td>weighting factor</td>
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<td>n/a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu_2$</td>
<td>weighting factor</td>
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<td>n/a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu_3$</td>
<td>weighting factor</td>
<td>1/1000</td>
<td>n/a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deformation Work (kJ)</td>
<td>222.97</td>
<td>231.91</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mass (g)</td>
<td></td>
<td>871.34</td>
<td>755.45</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Objective 1: $f_1(x) = \sum_{p=1}^{1000} (M^p_{\text{ref}} - M^p_{\text{cur}}(x))^2$ (N$^2$m$^2$)

Objective 2: $f_2(x) = |W_{\text{ref}} - W_{\text{cur}}(x)|$ (J)

Objective 3: $f_3(x) = m(x)$ (g)

minimise: $F(x) = (f_1(x), f_2(x), f_3(x))^T$

Design variables: $x = (w_1, w_2, s_1, s_2, s_3)^T \in \mathbb{R}$, $w_1, w_2 \in \mathbb{R}$, $s_1, s_2, s_3 \in \mathbb{Z}$

$f_0(x) = \nu_1 f_1(x) + \nu_2 f_2(x) + \nu_3 f_3(x)$

2.2.3. Design Variables

The stacking sequences $s_{1,2,3}$ under investigation are quasi-isotropic, balanced and symmetric. Following best practice stacking rules, the stacks range from 8 plies (1.336 mm) up to 26 plies (4.342 mm). The stacks with the range specified are generic but of similar scale as reported in Woodson [15] and Heimbs et al. [3]. The individual layer orientations within the stacks are either $0^\circ$, $45^\circ$, $90^\circ$ or $135^\circ$. Thus, the layer orientations within stacking sequence 0 are $[45,135,0,90]$, while for stacking sequence 26 they are $[45,135,0,90,0,45,135,0,90,0,45,135,0]$. The stacking sequences 0 and 26 act as lower and upper boundary respectively. In between the two, the stack thickness increases gradually with increasing sequence number. For stacks with identical thickness, the layer orientation of plies at the symmetry plane was varied to achieve 26 different generic stacking sequences. The list of stacking sequences is provided to the optimisation framework in an ascending order. The geometric design variables affect the width of the upper and lower flange of the z-frame, see figure 1(c). Both design variables range from 14.00 mm up to 28.00 mm, which is a generic specification similar to a typical z-frame design. [4]
3. Results and Discussion

Applying the optimisation algorithm on the optimisation task specified in equation (2), the following results are obtained. Figure 3 provides the set of 100 Pareto optimal solutions received from the optimisation after 20,000 function evaluations, which was set as the stop criterion. This amount of function evaluations was selected as a compromise between the exploration of the solution space and the overall computational time. In order to display the pareto optimal solutions within the solution space, the three objectives are normalised individually by the maximal values supplied for the respective objective from the entire set of function evaluations. A single best solution is identified by the application of equation (3), which is the weighted sum of the individual three objectives. It is indicated by a white cross in the bottom left of the solution space.

The single best solution is examined in more detail. Figure 4 depicts the structural response of the z-frame. The moment versus rotation graph provides information on the deformation history of the z-frame. While the bending moment was extracted via section forces at l/2 which marked the section centre, the rotation was obtained at the free end of the section. Since explicit calculations were executed, the analysis was based on stress wave propagation. Therefore, the bending moment was initially zero while the rotation started increasing. Due to the inertia of the section and the related structural dynamics of the response, the bending moment exhibits negative values. Up until the first peak at about 0.037 rad rotation, pure bending was the dominating deformation mode. From this point onwards, the central cross section of the z-frame started to rotate as the bending deformation increased. The flexural-torsional buckling behaviour was triggered. A second load peak formed at about 0.079 rad rotation. Afterwards, the load decreased until the onset of structural failure, which is a similar structural response as found in Weiß et al. [4]. Due to the nature of primarily elastic deformation as well as the limits on the design variables, a certain deviation between the best identified response curve and the desired reference curve remains. Improved equality between the two curves can hardly be achieved with the current parameter
Figure 4. Dynamic response curve of best solution applying equation (3) in comparison to the desired reference curve and to the best solution obtained from Weiß et al. [4]

specification. In addition to the limits of the design variables of the optimisation model, the exclusive consideration of a z-shape type frame design restricts the search for increased equality. Most notably, a constant laminate stacking along the frame span is preventing a better alignment with the desired structural behaviour.

Figure 4 contains an additional best solution obtained from a previous study [4]. It used the same numerical model and settings of the optimisation framework as outlined above. Besides the application of a different evolutionary algorithm, the major difference is in the formulation of the optimisation model. While the optimisation model of the present paper aims to identify a response curve most suitable to map onto a reference curve, the previous approach aimed to identify the most efficient z-frame design without the correlation to a reference curve. The measure of efficiency was defined as the elastic deformation work per unit mass, which was subject to maximisation. In contrast to the identification of a best response curve, the previous approach represents the identification of a response value. The reformulation of the optimisation problem towards a response curve identification suggests different z-frame design parameters which ultimately lead to an increased equality between the reference and the obtained response curve. However, the difference between the two responses becomes obvious from 0.07 rad rotation onwards. Prior to this point, the response curves are almost identical. This situation is induced by the specification of the reference curve. By chance, the reference curve, which is the result of a preparative study, and the response curve of the previous optimisation approach are already of reasonable equality.

Table 1 provides further information on the best design and compares it to the design obtained from the optimisation approach considered in Weiß et al. [4].

4. Conclusions

In this paper a heuristic optimisation approach is proposed which aims to identify a composite z-frame design being most suitable to achieve a specific dynamic response curve. Due to the choice of non-relaxable integer type and continuous type design variables, which relate to discrete laminate stacks and geometric dimensions of the z-frame respectively, the optimisation task was nonlinear. Therefore, the
mixed-integer distributed ant colony algorithm (MIDACO) was selected as the optimisation algorithm. The optimisation task was unconstrained and multi-objective. Based on weighting factors favouring the equality to the reference curve over the total amount of deformation work or the frame mass, a best solution was identified. The dynamic response curve of the best solution was compared to the reference curve as well as to the best solution of a different optimisation approach. Formulating the optimisation model towards a response curve optimisation instead of a response value optimisation was found beneficial. Since equality to the reference curve was the favoured optimisation objective, the obtained z-frame design was the best compromise between dynamic response, overall deformation work and frame mass. All in all, applying MIDACO for the optimisation of composite structures, which is essentially a mixed-integer type of problem, proved it’s worth and is highly recommended. Especially using the open source programming language Python appeared beneficial with respect to the overall effort necessary to solve the given task.

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References