Altruistic Coalition Formation in Cooperative Wireless Networks

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***Abstract*—In this paper, altruistic coalition formation in cooperative relay networks is studied. The communication is performed over two phases, the broadcasting phase and the cooperation phase. In the broadcasting phase, each node broadcasts its signal in its time-slot, while in the cooperation phase, all the nodes within their coalitions simultaneously relay each others' signals. A distributed merge-and-split algorithm is proposed to allow nodes to form coalitions and improve their total achievable rate. Moreover, the impact of different power allocation criteria is studied, where the sum-of-rates maximizing power allocation is shown to promote altruistic coalition formation and results in the largest coalitions among the different power allocation criteria. Finally, the proposed algorithm is compared with centralized power allocation and coalition formation, and shown to yield a good tradeoff between network sum-rate and computational complexity.**

***Index Terms*—Coalition formation, cooperation, decode-and-forward (DF), network coding, power allocation.**

I. INTRODUCTION

IN ad-hoc wireless networks, network nodes are independent, autonomous and selfish by nature and thus may not voluntarily share their transmission resources with other nodes. In other words, there is an element of competition and selfishness since all participating network nodes desire to maximize their utilities by maximizing their share of transmission resources. Moreover, randomly distributed nodes with local information may not know whom to cooperate with even if they are willing to cooperate. Although cooperative communications have been shown to yield significant performance gains [1], cooperation entails several costs, such as bandwidth and power. Ignoring such costs is unwarranted as it may severely affect the nodes' own performance. Particularly, network nodes may not cooperate and instead divert their resources to direct data transmissions. Alternatively, a group of nodes could form a coalition and cooperate to maximize the overall gains of the group and thus promote altruism. Specifically, each node seeks partners to form a cooperative coalition to achieve rate improvement for itself and/or for the whole coalition. In such a case, the greatest immediate benefits may not be achieved by the nodes that bear the greatest costs. Establishing cooperation in wireless ad-hoc networks without a centralized controller is a dynamic process. Hence, designing practical distributed algorithms that can promote cooperation without relying on centralized control is a highly desirable but considerably difficult task.

Coalitional game theory has emerged as an effective mathematical tool for modeling users' cooperation and designing distributed protocols in wireless networks. Several works have considered coalitional formation for user cooperation in wireless networks. For instance, a simple and distributed merge-and-split algorithm is proposed in [2] for the formation of virtual multiple input multiple output (MIMO) clusters of selfish single-antenna nodes. In [3], the stability of the grand coalition of transmitter and receiver cooperation in an interference channel is studied for both flexible transferable and non-transferable apportioning schemes. The curse of the boundary nodes in selfish packet-forwarding wireless networks is resolved using coalitional games in [4]. In [5], fair group coalitions for power-aware routing in wireless networks is studied and distributed algorithms based on max-min fairness are proposed. Distributed coalitional formations with transferable utilities and stable outcomes in relay networks are studied in [6]. In [7], a coalition formation game for relay transmission is studied based on a Markov-chain model, where the nodes are assumed to be selfish and aim at maximizing their individual throughput. Furthermore, stability analysis of the coalition formation process under system parameters' uncertainty is also investigated. A distributed coalition formation strategy for collaborative sensing tasks in camera sensor networks has been studied in [8,9]. Specifically, the authors propose a model that supports task-driven node selection and aggregation that is based on local decision-making and inter-node communication, and provides robustness and scalability.

In this work, altruistic coalition formation is particularly considered and the aim is to address the following questions: (1) How can coalitions be formed in a distributed fashion?, (2) What is the impact of different power allocation criteria on coalition formation?, and (3) What is the effect of mobility on the coalition formation process? To form cooperative groups, a coalition formation algorithm based on merge-and-split rules is proposed and proven to converge with arbitrary merge-and-split iterations. Each network node is treated as a player, who seeks partners to form a cooperative group to improve its transmission rate and/or that of the whole group through spatial diversity while incurring some power cost to meet a target SNR for information exchange. This in turn suggests a tradeoff between the gains and costs of cooperation. Furthermore, as the size of the cooperative group increases, both the gain and the cost also increase to the point where adding an additional node results in a cost that outweighs

The associate editor coordinating the review of this paper and approving it for publication was Prof. H. Li. Manuscript received September 11, 2012; revised February 5, May 25, July 27 and September 25, 2013.

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Digital Object Identifier 10.1109/TCOMM.2013.09.130519

0090-6778/10$25.00 © 2013 IEEE

the diversity gain. Additionally, since network nodes can be either mobile or static, the aim here is to analyze the long-term behavior of network coalition formation when the nodes are mobile within the network area as opposed to their coalition formation when they are static (i.e. at fixed locations). To the best of the authors' knowledge, no existing work has employed coalitional games in the analysis and design of algorithms for altruistic coalition formation in network-coded cooperative wireless networks[1].

The main contributions of this paper are summarized as follows:

- Modeled network-coded cooperative transmissions as a coalitional game in partition form with non-transferable utility and preference relation based on utilitarian order.
- Proposed a distributed merge-and-split algorithm for altruistic coalition formation in ad-hoc wireless networks. Additionally, the convergence, partition stability and complexity properties of the proposed distributed algorithm have been studied.
- Evaluated the impact of different power allocation criteria on coalition formation, where the sum-of-rates maximizing power allocation has been shown to promote altruistic coalition formation and result in the largest coalitions among the different power allocation criteria.
- Evaluated the proposed algorithm and demonstrated that it can achieve a network sum-rate that is comparable with that of a centralized algorithm; but with less computational complexity. Additionally, it has been shown to efficiently adapt to nodes' mobility and network topology changes.

In the remainder of this paper, the system model is presented in Section II. In Section III, the coalitional formation framework is discussed, while the proposed distributed coalition formation algorithm is provided in Section IV. The different cooperative power allocation criteria are discussed in Section V, while the convergence, stability and complexity properties of the proposed distributed algorithm are discussed in Section VI. The numerical results are presented in Section VII, while Section VIII discusses some related practical issues. Finally, conclusions are drawn in Section IX.

II. System Model

Consider an ad-hoc wireless network that consists of N single-antenna half-duplex decode-and-forward nodes which are denoted $S_1, S_2, \ldots, S_N$ for $N \geq 3$. Each node wishes to exchange its data symbol x_j for $j \in \{1, 2, \ldots, N\}$ with a common destination node D. The channel between nodes S_j and S_i is given by $h_{j,i} = e^{\jmath\theta_{j,i}}\sqrt{d_{j,i}^{-\nu}}$, with ν being the path-loss exponent and $\theta_{j,i}$ is the signal's phase uniformly distributed in the interval $[0, 2\pi]$ while $d_{j,i}$ is the distance between the two nodes [2]. Also, the channel $h_{j,i}$ between nodes S_j and S_i is assumed to be reciprocal (i.e. $h_{j,i} = h_{i,j}$) with perfect channel estimation at each node.

[1]The material in this paper has been presented in part at the 8th IEEE International Conference on Wireless and Mobile Computing, Networking and Communications (WiMob), Oct. 2012 [10]. In this correspondence, the system model is extended, convergence and stability properties of the proposed coalition formation algorithm are thoroughly studied and the effect of mobility on coalition formation is considered.

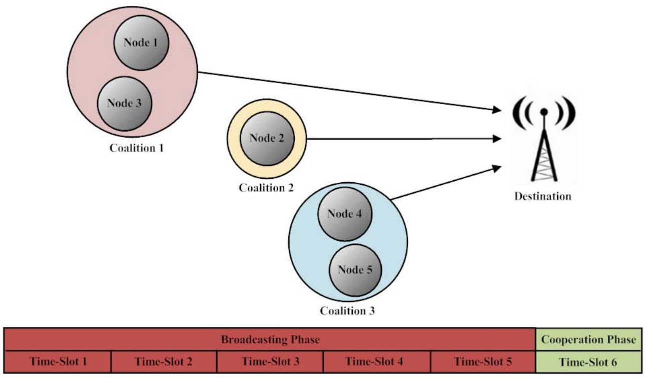

Fig. 1. Example of Cooperative Coalitions and Their Transmissions

Further, let $\mathcal{S} = \{S_1, S_2, \ldots, S_N\}$ be the finite, non-empty set of all network nodes that eventually self-organize into K (for $1 \leq K \leq N$) mutually exclusive coalitions of cooperative nodes $\mathcal{C} = \{C_1, C_2, \ldots, C_K\}$ with no cooperation between coalitions. Also, let $C_k \subseteq \mathcal{S}$ denote a coalition with $|C_k|$ nodes (where $|\cdot|$ represents the cardinality of a set) and $1 \leq |C_k| \leq N$. An individual non-cooperative player is called a singleton coalition while the set $\mathcal{S}$ is called the grand coalition when all the N network nodes cooperate. An example of a network of $N = 5$ nodes with a possible coalition formation is illustrated in Fig. 1. The communication between each node and the destination is performed in a TDMA fashion over $N + 1$ time-slots and is split into two phases, namely the broadcasting phase and the cooperation phase.

A. Broadcasting Phase

In the broadcasting phase of N time-slots, each node S_j—in its time-slot T_j—broadcasts its data symbol x_j, which is received by the $N - 1$ other nodes S_i in the network for $i \in \{1, 2, \ldots, N\}_{i \neq j}$, and the destination. The signal received at node S_i for $i \neq j$ is expressed as

$$y_{j,i} = \sqrt{P_{B_j}} h_{j,i} x_j + n_{j,i}, \tag{1}$$

while the received signal at the destination is given by

$$y_{j,d} = \sqrt{P_{B_j}} h_{j,d} x_j + n_{j,d}, \tag{2}$$

where P_{B_j} is the transmit power in the broadcasting phase at node S_j, and $n_{j,i}$ and $n_{j,d}$ are zero-mean complex additive white Gaussian noise (AWGN) samples with variance N_0, at node S_i and the destination, respectively. Upon the completion of the broadcasting phase, each node S_i will have received a set of $N - 1$ signals $\{y_{j,i}\}_{j=1,j\neq i}^{N}$ comprising symbols $x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_N$ from all the other nodes in the network. In addition, the destination will have received N signals $\{y_{j,d}\}_{j=1}^{N}$. Each node S_i then performs a matched filtering operation on each of the received signals $y_{j,i}$ and the signal-to-noise ratio (SNR) at the output of the matched-filter is given by [1]

$$\gamma_{j,i}^{BP} = \frac{P_{B_j} |h_{j,i}|^2}{N_0} = \frac{P_{B_j} d_{j,i}^{-\nu}}{N_0}. \tag{3}$$

Let f_j denote the node farthest from S_j in S_j's coalition C_k. Each node $S_j \in C_k$ broadcasts its symbol using transmit power P_{B_j} required to maintain a target SNR of γ between itself and node S_{f_j} as

$$P_{B_j} \geq \gamma N_0 d_{j,f_j}^{\nu}, \quad (4)$$

where it is assumed that γ is common to all the nodes in the network. Clearly, there is a tradeoff between the power invested in satisfying the target SNR and power allocated to the other members in coalition C_k. It should be noted that each node has a transmit power constraint of P that is shared between the two transmission phases as given by $P = P_{B_j} + P_{C_j}$, where P_{C_j} is the effective cooperative power at node S_j to relay the symbols of the other nodes in coalition C_k. Specifically, $P_{C_j} = \max\left[0, \min(P - P_{B_j}, P)\right]$, with $P_{C_j} = \sum_{S_i \in C_k, i \neq j} P_{C_{i,j}}$, and $P_{C_{i,j}}$ is the cooperative power node S_j utilizes in relaying node S_i's symbol x_i to the destination, for $i \neq j$.

B. Cooperation Phase

In the cooperation phase, each node S_i for $S_i \in C_k, \forall C_k \in \mathcal{C}$ and $|C_k| \geq 2$ in time-slot T_{N+1} forms a linearly-coded signal $\mathcal{X}_i(t)$ of the $|C_k| - 1$ received signals from the nodes in C_k, during the broadcasting phase. For multiuser detection at the destination, each decoded symbol x_l at node S_i is spread using a signature waveform $c_l(t)$, where it is assumed that each node S_i (for $i \neq l$) and the destination know the signature waveforms of all the other nodes in the coalition. The cross-correlation of $c_l(t)$ and $c_i(t)$ is $\rho_{l,i} = \langle c_l(t), c_i(t)\rangle \triangleq (1/T_s)\int_0^{T_s} c_l(t)c_i^*(t)dt$ for $l \neq i$ with $\rho_{i,i} = 1$ and T_s being the symbol duration. Thus, the transmitted signal by node S_i is

$$\mathcal{X}_i(t) = \sum_{S_l \in C_k, l \neq i} \sqrt{P_{C_{l,i}}} x_l c_l(t). \quad (5)$$

The received signal at the destination—assuming perfect timing synchronization—is written as

$$\mathcal{Y}_d(t) = \sum_{i=1}^{N} h_{i,d}\mathcal{X}_i(t) + n_d(t), \quad (6)$$

where $n_d(t)$ is the AWGN process at the destination. By substituting (5) into (6), $\mathcal{Y}_d(t)$ is re-written as

$$\mathcal{Y}_d(t) = \sum_{k=1}^{K}\sum_{S_l \in C_k}\left(\sum_{S_i \in C_k, i \neq l}\sqrt{P_{C_{l,i}}}h_{i,d}\right)x_l c_l(t) + n_d(t). \quad (7)$$

At the destination, multiuser detection is performed on $\mathcal{Y}_d(t)$ to extract symbol x_j of node $S_j \in C_k, \forall C_k \in \mathcal{C}$. Specifically, $\mathcal{Y}_d(t)$ is passed through a matched filter bank (MFB), yielding

$$\mathcal{Y}_{j,d} = \langle \mathcal{Y}_d(t), c_j(t)\rangle = \sum_{k=1}^{K}\sum_{S_l \in C_k}\alpha_l x_l \rho_{l,j} + \bar{n}_{j,d}, \quad (8)$$

where $\bar{n}_{j,d}$ is a zero-mean AWGN noise sample with variance N_0, while α_l is given by

$$\alpha_l = \sum_{S_i \in C_k, i \neq l}\sqrt{P_{C_{l,i}}}h_{i,d}. \quad (9)$$

The output of the MFB is expressed in vector form of N signals as $\boldsymbol{Y}_d = \boldsymbol{R}\boldsymbol{A}\boldsymbol{x} + \bar{\boldsymbol{n}}_d$, where $\boldsymbol{Y}_d = \left[\mathcal{Y}_{1,d}, \ldots, \mathcal{Y}_{N,d}\right]^T$, $\boldsymbol{x} = \left[x_1, \ldots, x_N\right]^T$, and $\bar{\boldsymbol{n}}_d = \left[\bar{n}_{1,d}, \ldots, \bar{n}_{N,d}\right]^T \sim \mathcal{CN}\left(0, N_0\boldsymbol{R}\right)$. Also, $\boldsymbol{R}$ and $\boldsymbol{A}$ are $N \times N$ matrices, where $\boldsymbol{R}$ is given by

$$\boldsymbol{R} = \begin{bmatrix} 1 & \rho_{1,2} & \cdots & \rho_{1,N} \\ \rho_{2,1} & 1 & \cdots & \rho_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{N,1} & \rho_{N,2} & \cdots & 1 \end{bmatrix}, \quad (10)$$

while the diagonal matrix $\boldsymbol{A}$ is expressed as $\boldsymbol{A} = \text{diag}\left[\alpha_1, \alpha_2, \ldots, \alpha_N\right]$. The received signal vector $\boldsymbol{Y}_d$ is then decorrelated (assuming matrix $\boldsymbol{R}$ is nonsingular) as $\tilde{\boldsymbol{Y}}_d = \boldsymbol{R}^{-1}\boldsymbol{Y}_d = \boldsymbol{A}\boldsymbol{x} + \tilde{\boldsymbol{n}}_d$, where $\tilde{\boldsymbol{n}}_d = \boldsymbol{R}^{-1}\bar{\boldsymbol{n}}_d$ and $\tilde{\boldsymbol{n}}_d \sim \mathcal{CN}\left(\boldsymbol{0}, N_0\boldsymbol{R}^{-1}\right)$. It is assumed that $\rho_{l,j} = \rho, \forall l \neq j$. Therefore, the decorrelated received signal is obtained as

$$\tilde{\mathcal{Y}}_{j,d} = \alpha_j x_j + \tilde{n}_{j,d}, \quad (11)$$

where $\tilde{n}_{j,d} \sim \mathcal{CN}\left(0, N_0\varrho_N\right)$, and ϱ_N is given by

$$\varrho_N = \frac{1 + (N-2)\rho}{1 + (N-2)\rho - (N-1)\rho^2}. \quad (12)$$

The received instantaneous SNR of node S_j's symbol (where $S_j \in C_k$) at the destination is given by $\gamma_j = \gamma_j^{BP} + \gamma_j^{CP}$, where γ_j^{BP} is expressed in (3), and γ_j^{CP} is obtained by passing $\tilde{\mathcal{Y}}_{j,d}$ through a matched-filter. Therefore, γ_j is obtained as [11]

$$\gamma_j = \frac{P_{B_j}|h_{j,d}|^2}{N_0} + \sum_{S_i \in C_k, i \neq j}\frac{P_{C_{j,i}}|h_{i,d}|^2}{N_0\varrho_N}. \quad (13)$$

Upon the completion of the broadcasting and cooperation phases, the destination will have received $|C_k|$ independent copies of symbol x_j of node $S_j \in C_k$ and thus achieving a diversity order of $|C_k|$ [1].

III. Coalition Formation Framework

Let $\upsilon_j(C_k)$ denote the payoff of each node S_j in coalition C_k, which is assumed to be equal to the achievable rate. Based on the discussed system model, a singleton coalition of node S_j occurs when it does not form a cooperative coalition with other nodes. In this case, node S_j utilizes all its available power P and transmits its data once every $N+1$ time-slots. Thus, the payoff of node S_j is obtained as

$$\upsilon_j(\{S_j\}) = R_{j,d}^{D} = \frac{1}{N+1}\log_2\left(1 + \frac{P|h_{j,d}|^2}{N_0}\right), \quad (14)$$

where $R_{j,d}^{D}$ is the achievable rate with direct transmission. Additionally, $R_{j,d}^{D}$ represents the non-cooperative payoff of any node S_j for $S_j \in \mathcal{S}$. On the other hand, for coalition C_k with $|C_k| \geq 2$, the achievable rate of node S_j due to the cooperative transmission is given by

$$R_{j,d}^{C} = \frac{1}{N+1}\log_2\left(1 + \frac{P_{B_j}|h_{j,d}|^2}{N_0} + \sum_{S_i \in C_k, i \neq j}\frac{P_{C_{j,i}}|h_{i,d}|^2}{N_0\varrho_N}\right). \quad (15)$$

Therefore, the payoff of each node S_j in coalition C_k is given by $\upsilon_j(C_k) = R_{j,d}^{C}, \forall S_j \in C_k$. Consequently, the value of a coalition is

$$\upsilon(C_k) = \sum_{S_j \in C_k}\upsilon_j(C_k), \quad (16)$$

which is equivalent to the sum-rate of the coalition.

Definition 1: A coalition game is said to have a non-transferable utility (NTU) if the coalition value cannot be arbitrarily apportioned among its nodes and each node will have its own value within a coalition.

$$\mathcal{V}(C_k) = \left\{ \boldsymbol{\upsilon}(C_k) \in \mathbb{R}^{|C_k|} \mid \forall S_j \in C_k, \upsilon_j(C_k) = R^C_{j,d} \geq 0, \text{ if } P_{C_j} > 0, \text{ and } \upsilon_j(C_k) = -R^D_{j,d}, \text{ otherwise.} \right\}. \tag{17}$$

Based on the proposed system model, the coalition game in hand has a non-transferable utility, as a specific achievable rate for each node in a coalition is achieved [12]. In addition, the coalition game formulated in this work is in the characteristic function form. That is, utilities achieved by the players in a coalition are unaffected by those outside it.

Definition 2: A coalitional game with non-transferable utility is defined by a pair $(\mathcal{S}, \mathcal{V})$, where $\mathcal{S}$ is a finite set of N players, and $\mathcal{V}$ is a set valued function such that for every coalition $C_k \subseteq \mathcal{S}$, $\mathcal{V}(C_k)$ is a closed convex subset of $\mathbb{R}^{|C_k|}$ that contains the payoff vectors the players in C_k can achieve.

In the proposed system model, $\mathcal{V} : C_k \to \mathbb{R}^{|C_k|}$ such that $\mathcal{V}(\phi) = \phi$, and if $C_k \neq \phi$, then $\mathcal{V}(C_k)$ is non-empty and closed. Moreover, the coalitional set-valued function $\mathcal{V}$ of a coalition $C_k \subseteq \mathcal{S}$ is defined as in (17) . Note that $P_{C_j} = 0$ if and only if $P_{B_j} = P$, which implies that node S_j has no interest in cooperation.

Remark 1: The proposed network-coded transmission model is a coalitional game $(\mathcal{S}, \mathcal{V})$ in partition form with non-transferable utility, where $\mathcal{V}(C_k)$ is a singleton set, as defined by (17), and is thus a closed and convex subset of $\mathbb{R}^{|C_k|}$.

Remark 2: In the proposed NTU coalitional game $(\mathcal{S}, \mathcal{V})$, the grand coalition of all the nodes rarely forms due to the target SNR power costs.

Based on the SNR target defined in (4), the power cost for coalition formation depends on the distance between the network nodes which in turn governs coalitions' sizes. As the size of a coalition increases, the cooperative gain and power cost per node also increase (i.e. cooperation gains in a coalition are limited by power costs). However, the power saving due to the diversity gains gradually diminish even with the increase in the number of cooperative nodes in a coalition, at which point, no additional nodes should join the coalition. This prevents cooperative networks from forming a grand coalition and instead form independent disjoint coalitions. In turn, the proposed game is modeled as a coalition formation game, with the aim of finding the network's coalitional structure [12]. It is noteworthy that solution concepts for coalitional games based on the core are not applicable to the game model in hand due to the power costs.

Additionally, it should be noted that power costs at each node forming a coalition are based on the following. First, to cooperate, the node must allocate power to the cooperation phase, which limits the transmit power available for broadcasting of node's own symbols. Second, cooperation with the nodes in a given coalition imposes a lower-bound on the broadcasting power (to ensure target SNR is met within the coalition).

IV. Design of Distributed Coalition Formation Algorithm

In this section, the aim is to study how coalitions can be formed in a distributed manner. Specifically, a coalition is formed if it is beneficial to at least one node in the coalition and also for the coalition as a whole. Also, the nodes of a coalition can avoid merging with other coalitions if they are as well off as a result of not merging. Furthermore, when nodes form a coalition, they cannot unilaterally deviate on their own. In turn, coalition structure changes are determined by the members of a coalition interacting with one another as a unit.

Since network nodes are rational and autonomous, the design of an iterative distributed algorithm to form a network coalition structure that improves that network sum-rate is highly desirable. But first, several concepts must be defined.

Definition 3: A collection of coalitions, denoted as $\mathcal{C}$, is defined as $\mathcal{C} = \{C_1, C_2, \ldots, C_K\}$ for $2 \leq K \leq N$ mutually disjoint coalitions C_k of $\mathcal{C}$. Equivalently, a collection is any arbitrary group of disjoint coalitions C_k of $\mathcal{C}$ that does not necessarily span all the players of $\mathcal{S}$. If a collection spans all the players in $\mathcal{S}$ (i.e. $\bigcup_{k=1}^{K} C_k = \mathcal{S}$), the collection is a partition of $\mathcal{S}$.

Definition 4: A preference operator $\triangleright$ is defined for comparing two collections $\mathcal{Q} = \{Q_1, \ldots, Q_l\}$, and $\mathcal{R} = \{R_1, \ldots, R_p\}$ that are partitions of the same subset $\mathcal{A} \subseteq \mathcal{S}$ (i.e. same players in $\mathcal{Q}$ and $\mathcal{R}$). Thus, $\mathcal{Q} \triangleright \mathcal{R}$ implies that the way $\mathcal{Q}$ partitions $\mathcal{A}$ is preferred to the way $\mathcal{R}$ partitions $\mathcal{A}$.

In the coalitional game theory literature, comparison relations based on orders are split into two categories [13]: individual value orders and coalition value orders. In the former category, comparison is performed on the basis of individual payoffs (e.g. Pareto order). Specifically, no node is willing to form a coalition with other nodes if forming that coalition would result in a transmission rate that is less than its direct transmission rate (i.e. selfish behavior). In the latter category, two collections (or partitions) are compared based on the value of the coalitions inside these collections, such as the utilitarian order (e.g. $\mathcal{Q} \triangleright \mathcal{R} \Longrightarrow \sum_{i=1}^{l} \upsilon(Q_i) > \sum_{i=1}^{p} \upsilon(R_i)$). Particularly, a node will form a coalition with other nodes even if its achievable cooperative transmission rate is less than its direct transmission rate, as long as the sum-rate of the coalition is improved and is greater than the sum of the direct transmission rates of all the nodes within the coalition (i.e. altruistic behavior). In this work, the utilitarian order comparison relation is assumed as it is more suited to the studied altruistic coalition formation.

There are two successive rules for forming and breaking coalitions, known as merge-and-split rules [13].

Definition 5 (Merge Rule): Merge any collection of disjoint coalitions $\{Q_1, \ldots, Q_l\}$, where $\{\bigcup_{k=1}^{l} Q_k\} \triangleright \{Q_1, \ldots, Q_l\}$, thus $\{Q_1, \ldots, Q_l\} \to \{\bigcup_{k=1}^{l} Q_k\}$.

Definition 6 (Split Rule): Split any coalition $\{\bigcup_{k=1}^{l} Q_k\}$, where $\{Q_1, \ldots, Q_l\} \triangleright \{\bigcup_{k=1}^{l} Q_k\}$, thus $\{\bigcup_{k=1}^{l} Q_k\} \to \{Q_1, \ldots, Q_l\}$.

The merge-and-split rules simply mean that two (or more) coalitions will merge if their merger would do more good than harm to the overall coalition value (or equivalently, sum-rate) of the merged coalition. Otherwise, coalitions will split into smaller ones or even singletons.

As noted earlier, most cooperative communication systems proposed in the literature are based on the assumption of fully cooperative behaviors; while ignoring cooperation costs. Such costs not only limit the benefits of cooperation but also impair the user's performance. Additionally, most of the current research consider selfish nodes; however, in this work, the case of altruistic nodes that aim at improving the network sum-rate is considered. Therefore, there is a need for deriving a practical distributed algorithm that promotes cooperation, takes into account the tradeoff between cooperation gains and costs, and eliminates the need for centralized entities (as in the case of ad-hoc wireless networks).

Due to the fact that network nodes are geographically distributed, then nodes must be able to self-organize into a stable network partition and adapt this structure to topology changes and mobility, even in the absence of a centralized controller. For our coalitional game-theoretic algorithmic design, the utility of each node has been defined as its achievable rate, and cooperation costs have been taken into account in the form of the transmit power need to satisfy a target SNR each neighboring potential node. Additionally, the novel concept of merge-and-split will be utilized to allow network nodes to form altruistic coalitions based on the utilitarian order. Specifically, the merge-and-split processes will be based on the sum-rate improvement achievable through altruistic coalition formation.

A. Algorithm Description

The network operation starts at $\tau = 0$ with network nodes being partitioned into singleton coalitions (i.e. $C_j = \{S_j\}$ for $1 \leq j \leq N$ such that $\mathcal{C} = \{\{S_1\}, \{S_2\}, \ldots, \{S_N\}\}$) and each node S_j determines its achievable direct transmission rate $R^D_{j,d}$. After that, the following three phases take place.

1) Node Discovery: Each node $S_j \in \mathcal{S}$ discovers the neighboring potential nodes with which it can possibly merge using $P_{B_j} = P$. Specifically, for each node S_j, potential partners lie within a circle with radius determined by the power $P \geq \gamma N_0 d^{\nu}_{j,f_j}$ required for symbols' exchange while meeting the target SNR γ. Thus, if the received signal at node S_i satisfies γ, it is considered to be decoded correctly. Let $\mathcal{D}_j$ be the set of nodes that decoded node S_j's symbol correctly, i.e.

$$\mathcal{D}_j = \{\forall S_i \in \mathcal{S} \text{ and } i \neq j : \gamma^{BP}_{j,i} \geq \gamma\}. \tag{18}$$

After that, node S_j broadcasts a request-to-send (RTS) message which is received by all the nodes in $\mathcal{D}_j$. Then, each node $S_i \in \mathcal{D}_j$ replies to node S_j with a clear-to-send (CTS) message that contains its channel state information with the destination. If the decoding set of node S_j is empty (i.e. $\mathcal{D}_j = \phi$), then it employs direct transmission and does not form a coalition with any other node. Otherwise, node S_j enumerates all the possible distinct coalitions of $S_j \cup \mathcal{D}_j$. In the case of a coalition C_k, the potential nodes lie within the intersection of $|C_k|$ circles, each centered around node $S_j \in C_k$. Clearly, the node discovery phase significantly reduces the coalition formation space.

2) Adaptive Coalition Formation: In this phase, the time-index is updated to $\tau = \tau + 1$ and each node sequentially proposes to merge with one of its potential partners. If such a merge is desirable by all the nodes according to the utilitarian order, then a coalition with one or more of the potential nodes could form by a merge agreement of all the participating nodes. For all merged coalitions, a random node is elected as a coalition-head [14], which is responsible for periodically exchanging timing information with the rest of the coalition[2]. After that, the power allocation fractions of each node are determined according to one of the power allocation criteria discussed in Section V. After all the nodes have made their merge decisions, the merge process ends, resulting in a partition $\mathcal{M}^\tau = \textbf{Merge}(\mathcal{C}^{\tau-1})$.

If the sum-rate value a group of nodes achieved by forming a coalition is less than the value achieved before the merger, they split into singletons or coalitions of smaller sizes. At the end of the split process, a partition $\mathcal{C}^\tau = \textbf{Split}(\mathcal{M}^\tau)$ is obtained. A sequence of merge-and-split processes along with time-index updates take place in a distributed manner via appropriate control channels, depending on the achievable rate improvement of each node and coalition, until there is no need for any merging/splitting in the current partition, in which case the final partition $\mathcal{C}^* = \mathcal{C}^\tau$ is obtained.

It should be noted that in the node discovery phase, enumerating all distinct coalitions does not necessarily require significant computations. Even if the number of possible distinct coalitions is large, each node—in the adaptive coalition formation phase—takes turn in submitting merge proposals to other potential nodes. So, it is likely that other potential nodes may submit merge proposals. In other words, each node will not necessarily propose each possible coalition to its potential nodes as some of these nodes will propose them. Additionally, a node may not necessarily have to enumerate all possible distinct coalitions; on the contrary, a node might start enumerating only the small-sized potential coalitions and not have to propose larger coalitions if a sum-rate improvement is achieved with small coalitions. In other words, a complete enumeration of potential coalitions should not be required as the process can stop as soon as a candidate merger is identified.

3) Data Transmission: In this final phase, data transmission of each node takes place in the form of broadcasting and cooperation, over a total of $N + 1$ time-slots and as described in Section II. Finally, the above three phases repeat in response to topology changes or mobility, as discussed later. The network initialization and proposed distributed merge-and-split coalition formation algorithm are summarized in Table I.

It should be noted that the resulting partition from the proposed merge-and-split algorithm is not guaranteed to be optimal (i.e. is not the one that maximizes the network sum-rate). This is because the formed coalitions do not exchange information about their values and thus have no way of knowing whether there are different partitions that could lead to better network sum-rate. Even if all coalition values are known, no known algorithm can determine the optimal partition with time complexity that is polynomial in the number of possible coalitions [17].

V. Impact of Different Power Allocation Criteria

It is intuitive to note that network coalition formation is dependent on the cooperative power allocation within each coalition. Therefore, the following power allocation criteria are studied.

[2]The nodes' transmission within a coalition can be assumed to be perfectly synchronized during the cooperation phase [15] [16].

TABLE I
NETWORK INITIALIZATION AND PROPOSED DISTRIBUTED MERGE-AND-SPLIT COALITION FORMATION ALGORITHM

Initial State:
At the beginning of all time, initialize time-index at $\tau = 0$ with the network being partitioned as $\mathcal{C}^0 = \{\{S_1\}, \{S_2\}, \ldots, \{S_N\}\}$.
Coalition Formation Algorithm:
Phase 1 - Node Discovery:
Each node determines its neighbors and potential coalitions.
Phase 2 - Adaptive Coalition Formation:
Coalition formation using merge-and-split rules occurs.
repeat
(a) Update time-index: $\tau = \tau + 1$.
(b) $\mathcal{M}^\tau = \textbf{Merge}(\mathcal{C}^{\tau-1})$: coalitions in $\mathcal{C}^{\tau-1}$ make merge decisions based on the merge rule.
(c) $\mathcal{C}^\tau = \textbf{Split}(\mathcal{M}^\tau)$: coalitions in $\mathcal{M}^\tau$ make split decisions based on the split rule.
until merge-and-split terminates with final partition denoted $\mathcal{C}^*$.
Phase 3 - Data Transmission:
Each node transmits its data symbol in the broadcasting phase and the nodes within every coalition relay data for each other during the cooperation phase.

A. Equal Power Allocation (EPA)

Under this criterion, a node $S_i \in C_k$ determines its maximum required broadcasting power as $P_{B_i} = \max\{\gamma N_0/|h_{j,i}|^2\}_{S_j \in C_k, j\neq i}$ and then the cooperative power $P_{C_i} = P - P_{B_i}$ is equally allocated to the other nodes in C_k in the form of

$$\text{(EPA)}: \quad P_{C_{j,i}} = \frac{P - P_{B_i}}{|C_k| - 1}, \quad \forall S_j \in C_k, \text{ and } j \neq i. \tag{19}$$

In this case, each node naively allocates its remaining power equally to the other nodes in C_k.

B. Sum-of-Rates Maximizing Power Allocation (SRM-PA)

The sum-of-rates maximizing power allocation problem of coalition C_k is solved by the coalition-head and is expressed as

$$\text{(SRM-PA):} \qquad \max \quad \textstyle\sum_{S_i \in C_k} R_{i,d}^C$$

$$\text{s.t.} \quad P_{B_i} + \sum_{S_j \in C_k, j\neq i} P_{C_{j,i}} \leq P, \quad \forall S_i \in C_k, \tag{20a}$$

$$P_{B_i} \geq \gamma N_0/|h_{j,i}|^2, \qquad \forall S_j, S_i \in C_k \text{ and } j \neq i, \tag{20b}$$

$$P_{B_i} \geq 0, \qquad \forall S_i \in C_k, \tag{20c}$$

$$P_{C_{j,i}} \geq 0, \qquad \forall S_j, S_i \in C_k \text{ and } j \neq i. \tag{20d}$$

The first constraint in (20) enforces the total power constraint, while the second constraint ensures that the target SNR is met by each node $S_i \in C_k$. The last two constraints impose the non-negativity of the allocated broadcasting and cooperative powers, respectively.

C. Max-Min Rate Power Allocation (MMR-PA)

The power allocation problem under the max-min rate fairness criterion for coalition C_k solved by the coalition-head is expressed as

$$\text{(MMR-PA):} \qquad \max \quad \eta$$

$$\text{s.t.} \quad \eta - R_{i,d}^C \leq 0, \qquad \forall S_i \in C_k, \tag{21a}$$

$$P_{B_i} + \sum_{S_j \in C_k, j\neq i} P_{C_{j,i}} \leq P, \quad \forall S_i \in C_k, \tag{21b}$$

$$P_{B_i} \geq \gamma N_0/|h_{j,i}|^2, \qquad \forall S_j, S_i \in C_k \text{ and } j \neq i, \tag{21c}$$

$$P_{B_i} \geq 0, \qquad \forall S_i \in C_k, \tag{21d}$$

$$P_{C_{j,i}} \geq 0, \qquad \forall S_j, S_i \in C_k \text{ and } j \neq i. \tag{21e}$$

The first constraint imposes max-min rates while the rest of the constraints are as in problem (20). Problems SRM-PA and MMR-PA can be verified to be convex [18] and thus can be solved efficiently by any standard convex optimization algorithm [19]. Therefore, solving such problems at a coalition-head should pose no severe computational overhead.

Remark 3: Since the achievable rate of each node in a coalition C_k is strictly monotonically increasing in the allocated power, the total power constraint is always met (i.e. $P_{B_i} + \sum_{S_j \in C_k, j\neq i} P_{C_{j,i}} = P, \forall S_i \in C_k$ and $\forall C_k \in \mathcal{C}$).

VI. CONVERGENCE, PARTITION STABILITY AND COMPLEXITY

In this section, the convergence, partition stability, and complexity properties of the proposed algorithm are studied.

A. Convergence

We first establish the convergence of the coalition formation process. Note that similar results are proved in [13] and [20].

Theorem 1 (Convergence): The coalition formation process based on the merge-and-split rules converges in a finite number of iterations.

Proof: The merge-and-split rules under the utilitarian order only permit mergers and splits if they strictly improve the total value of the coalitions. Thus, the total value of the coalition partition is strictly increasing in the coalition formation process. Since the number of possible partitions is finite, the total value is bounded and the process must converge in a finite number of iterations. □

B. Partition Stability

Stable coalition structures in coalition formation games correspond to an equilibrium state in which users do not have incentives to leave the already formed coalitions. There are a number of possible notions of stability, and we follow [13] (and [20], which uses slightly different terminology) in characterizing these notions with respect to a *defection function*.

Definition 7 ($\mathbb{D}(\mathcal{P})$ Function): A defection function $\mathbb{D}$ is a function which associates with each partition $\mathcal{P}$ of $\mathcal{S}$ a group of collections in $\mathcal{S}$ [13].

In words, $\mathbb{D}(\mathcal{P})$ consists of all collections $\mathcal{C} = \{C_1, C_2, \ldots, C_l\}$ such that the players in $\mathcal{C}$, $\bigcup_{j=1}^{l} C_j$, can leave their current coalitions (specified by the partition $\mathcal{P}$) to form the new coalitions specified by $\mathcal{C}$.

Definition 8 ($\mathcal{C}$ in the frame of $\mathcal{P}$): Given a collection $\mathcal{C} = \{C_1, C_2, \ldots, C_l\}$ and a partition $\mathcal{P} = \{P_1, P_2, \ldots, P_k\}$ of $\mathcal{S}$, we define $\mathcal{C}$ in the frame of $\mathcal{P}$ as

$$\mathcal{C}[\mathcal{P}] = \left\{ P_1 \bigcap \bigcup_{j=1}^{l} C_j, P_2 \bigcap \bigcup_{j=1}^{l} C_j, \ldots, P_k \bigcap \bigcup_{j=1}^{l} C_j \right\} \setminus \emptyset. \tag{22}$$

That is, $C[P]$ is a collection consisting of the same elements as in the collection $\mathcal{C}$, divided according to the partition $\mathcal{P}$ [13].

Definition 9 ($\mathbb{D}$-Stability): A partition $\mathcal{P} = \{P_1, P_2, \ldots, P_k\}$ of $\mathcal{S}$ is $\mathbb{D}$-stable if $\mathcal{C}[\mathcal{P}] \triangleright \mathcal{C}$ for all $\mathcal{C} \in \mathbb{D}(\mathcal{P})$ such that $\mathcal{C}[\mathcal{P}] \neq \mathcal{C}$ [13].

In particular, we describe two important defection functions and their corresponding stability notions [13]. The $\mathbb{D}_c$ defection function associates with each partition $\mathcal{P}$ of $\mathcal{S}$ the group of all collections in $\mathcal{S}$. As such, this function allows any group of players to leave the partition $\mathcal{P}$ through *any* operation and create an arbitrary collection in $\mathcal{S}$. As a result of this complete flexibility, the resulting $\mathbb{D}_c$-stability is the strongest notion of stability.

Definition 10 ($\mathbb{D}_c(\mathcal{P})$ Function): For each partition $\mathcal{P}$, $\mathbb{D}_c(\mathcal{P})$ is the family of all collections of $\mathcal{S}$ [13].

The second defection function, $\mathbb{D}_{hp}$, associates with each partition $\mathcal{P}$ of $\mathcal{S}$ the group of all partitions of $\mathcal{S}$ that can form by merging or splitting the coalitions in $\mathcal{P}$.

Definition 11 ($\mathcal{P}$-Compatibility): A coalition T of $\mathcal{S}$ is called $\mathcal{P}$-compatible if for some $i \in \{1, \ldots, K\}$, $T \subseteq P_i$, and $\mathcal{P}$-incompatible, otherwise [13].

Definition 12 ($\mathcal{P}$-Homogeneity): A partition $\mathcal{Q} = \{Q_1, Q_2, \ldots, Q_l\}$ of $\mathcal{S}$ is called $\mathcal{P}$-homogeneous if for each $j \in \{1, \ldots, l\}$ there exists some $i \in \{1, \ldots, k\}$ such that either $Q_j \subset P_i$ or $P_i \subset Q_j$. So, any $\mathcal{P}$-homogeneous partition arises from $\mathcal{P}$ by allowing each coalition either to split into smaller coalitions or to merge with other coalitions (or to remain unchanged) [20].

Definition 13 ($\mathbb{D}_{hp}(\mathcal{P})$ Function): A function that associates with each partition $\mathcal{P}$ of $\mathcal{S}$ the family of all $\mathcal{P}$-homogeneous partitions of $\mathcal{S}$ [20].

Theorem 2 ($\mathbb{D}_{hp}$-Stability): A partition is $\mathbb{D}_{hp}$-stable if and only if it is the outcome of iterating the merge-and-split rules [20].

Proposition 1: The final partition $\mathcal{C}^*$ that results from our proposed algorithm is $\mathbb{D}_{hp}$-stable.

Proof: This is an immediate result of Theorem 2. □

Consequently, to find a $\mathbb{D}_{hp}$-stable partition, it suffices to iterate the merge-and-split rules starting from any initial network partition until partition $\mathcal{C}^*$ is reached, in which case players have no incentive to leave partition $\mathcal{C}^*$ through merge-and-split to form other partition in $\mathcal{S}$.

Theorem 3 ($\mathbb{D}_c$-Stability): If $\mathcal{P}$ is a $\mathbb{D}_c$-stable partition then

- $\mathcal{P}$ is unique, and
- $\mathcal{P}$ is the outcome of every iteration of the merge and split rules [13].

Proposition 2: The proposed coalition formation algorithm converges to the unique $\mathbb{D}_c$-stable partition, if such a partition exists. Otherwise, the proposed merge-and-split algorithm results in a $\mathbb{D}_{hp}$-stable network partition.

Proof: This is an immediate consequence of Theorem 3 and Proposition 1. □

Remark 4: If a $\mathbb{D}_c$-stable partition exists, then it will be the unique network sum-rate maximizing partition, and it will be the outcome of the merge-and-split algorithm starting from any initial network partition.

It is noteworthy that our simulation results suggest that usually no $\mathbb{D}_c$-stable partition exists (as will be further discussed in the penultimate paragraph of Section VII).

C. Complexity Analysis

1) Communication Complexity: The communication complexity of the proposed algorithm is related to the number of merge-and-split operations, which is directly related to the total number of coalition formation proposals $\mathcal{M}$ sent by each of the N nodes. Two extreme cases are considered: (1) if all the proposals are rejected, and (2) if all the proposals are accepted. In the first case and as described in Section IV-A, each node $S_i \in \mathcal{S}$ submits at most $|\mathcal{D}_i|$ proposals, where $|\mathcal{D}_i| \leq N-1$. Now, if the first node submits $N-1$ proposals and the second submits $N-2$ proposals and so on, then the total number of proposals is $\mathcal{M}_{\text{worst}} = \sum_{i=1}^{N-1} i = \frac{1}{2}N(N-1)$ [2]. Thus, in the worst case, the complexity is of the order $\mathcal{O}(N^2)$. In the second case where all the proposals are accepted, the total number of proposals is only $\mathcal{M}_{\text{best}} = N$, and a complexity order of $\mathcal{O}(N)$. In practice, the number of proposals is between these two extreme cases (i.e. $\mathcal{M}_{\text{best}} \leq \mathcal{M} \leq \mathcal{M}_{\text{worst}}$). In fact, the number of proposals is much lower than $\frac{1}{2}N(N-1)$ as the proposed algorithm tends to merge the smaller coalitions first and then the bigger ones but with reduced possibilities. Hence, if $\mathcal{L}$ messages are required per coalition formation proposal, then $\mathcal{L} \times \mathcal{M}$ messages are required until convergence of the algorithm.

2) Computational Complexity: An equally important factor into the operation of the distributed merge-and-split coalition formation algorithm is the computational complexity involved in the cooperative power allocation. As for the equal power allocation criterion, the calculation of power allocation at each node is trivial (i.e. with negligible computational complexity). However, for the sum-of-rates maximizing and max-min rate power allocation criteria, the computational complexity is dependent on the number of nodes in each coalition (which determines the number of variables and constraints). Despite the fact that such problems are convex and thus can be computed efficiently, doing so repetitively may impose significant overhead and delay to each coalition-head, especially for potentially large coalition sizes. In other words, the computational complexity is likely to increase with the size of the coalition as the complexity of a merge operation can grow significantly with the increase number of candidate nodes in the decoding set of each node. Thus, it takes longer to compute the value/sum-rate of a large coalition compared to a small coalition. However, due to the initial neighbor discovery phase and power costs, most network nodes tend to form coalitions of sizes less than $N/2$ even for dense networks, under the different power allocation criteria (as will be verified in the following section).

VII. Simulation Results

Consider an ad-hoc network with $N = 15$ nodes, where the network density varies with the square area of deployment. The path-loss exponent is set to $\nu = 3$, while the correlation coefficient is $\rho = 0.40$. The total power constraint per node is $P = 0.15$ W, while the noise variance is $N_0 = 10^{-5}$ W. The target SNR for information exchange is set to $\gamma = 3$ dB [1]. The simulation results are averaged over 10000 independent runs with the nodes randomly and uniformly distributed across

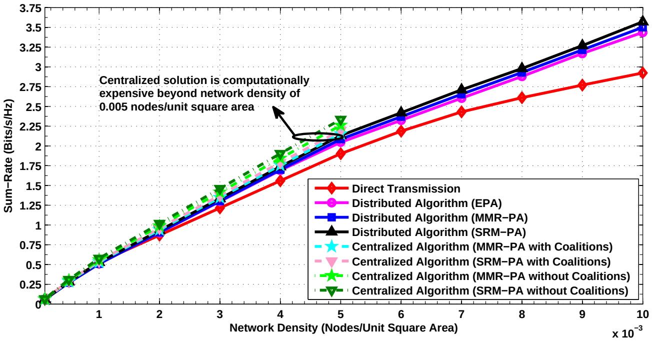

Fig. 2. Network Sum-Rate of Different Power Allocation Criteria

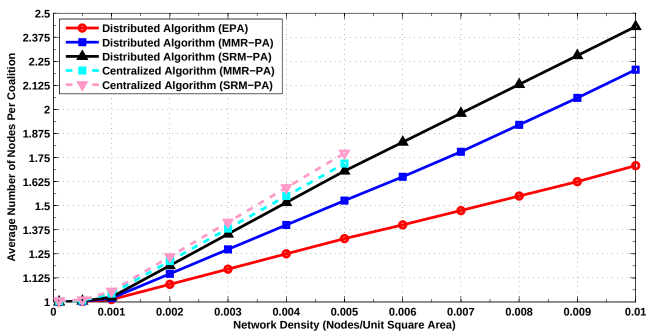

Fig. 3. Average Number of Nodes Per Coalition of Distributed and Centralized Algorithms

the deployment area for different network densities, while the destination is always located at the center of the area.

It is evident from Fig. 2 that as the network density increases, the network sum-rate under the different distributed and centralized power allocation criteria also increases and is superior to that of direct transmission[3]. This is because with the increase in network density for a fixed number of nodes, the deployment area decreases and the possibility of finding cooperative partners increases. Furthermore, the SRM-PA criterion achieves the highest sum-rate among the other power allocation criteria. Also, the centralized SRM-PA algorithm without coalition formation achieves the highest network sum-rate among all distributed and centralized power allocation criteria[4]. Moreover, the computational complexity of the centralized algorithms for network densities beyond 0.005 nodes/unit square area becomes extremely expensive[5].

As can be seen from Fig. 3, the SRM-PA criterion results in the the highest average number of nodes per coalition. This is due to the altruistic coalition formation and the fact that the SRM-PA criterion yields the highest sum-rate. Hence, network nodes tend to form larger coalitions under the SRM-PA criterion to improve the sum-rate of the coalition, which in turn reduces the average number of coalitions formed, as evident from Fig. 4.

[3]Direct transmission is performed over $N = 15$ time-slots with no cooperation phase.

[4]The mathematical formulations and complexity analysis of the centralized power allocation and coalition formation under different power allocation criteria can be found in [21, p. 152].

[5]The centralized MINLP power allocation problems are solved using MIDACO [22] with optimization tolerance set to 0.01.

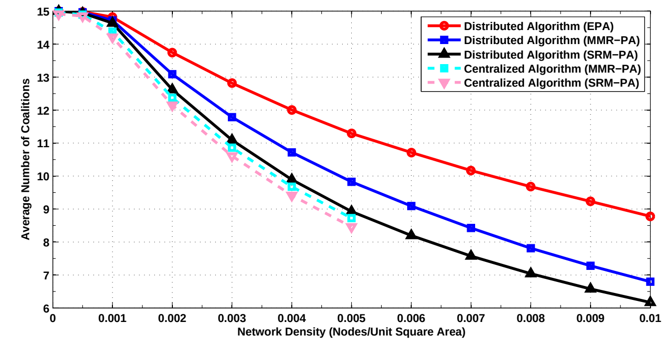

Fig. 4. Average Number of Coalitions of Distributed and Centralized Algorithms

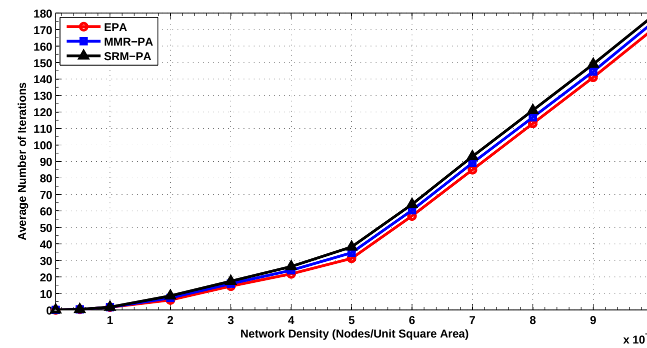

Fig. 5. Average Number of Iterations Under Different Power Allocation Criteria

In Fig. 5, the average number of iterations until convergence of the proposed distributed merge-and-split algorithm under the different power allocation criteria is shown. It can be seen that the SRM-PA criterion requires the largest number of iterations and this is because under this criterion, network nodes tend to form larger coalitions. Thus, in the proposed distributed merge-and-split algorithm, larger potential coalitions are formed and then possibly split, which in turn increases the number of iterations.

Based on the histogram shown in Fig. 6, it can be seen that a large portion of the nodes are participating in coalitions. Even for the EPA criterion, where singletons are most prevalent, more than half of the nodes are participating in coalitions of at least 2 nodes. As for the MMR-PA and SRM-PA criteria, more than half of the nodes are in coalitions of 3 or more nodes, with the SRM-PA criterion resulting in the largest coalitions.

In Fig. 7, the mobility of network nodes with density of 0.005 nodes/unit square area is evaluated, and the network operation is observed over a period of 4 minutes. The mobility of the individual nodes follows the random waypoint (RWP) mobility model [23]. The pause and motion times of each node under the RWP mobility model are uniformly distributed between $[0, 8]$ s, and $[2, 10]$ s, respectively. Moreover, the speed is uniformly distributed between $s_{\min} = 0.1$ m/s and $s_{\max}$, where $s_{\min}$ and $s_{\max}$ are the minimum and maximum speeds, respectively. As can be seen in Fig. 7, the average number of merge-and-split processes per minute increases with the increase in speed, as expected. This is because the higher is the speed, the more frequent are the network topology

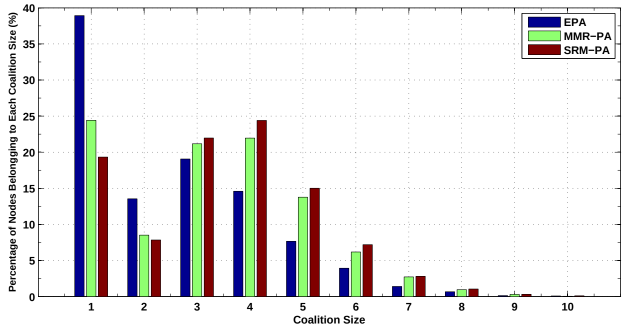

Fig. 6. Percentage of Nodes Belonging to Each Coalition Size Under the Proposed Distributed Algorithm - Network Density = 0.01 Nodes/Unit Square Area

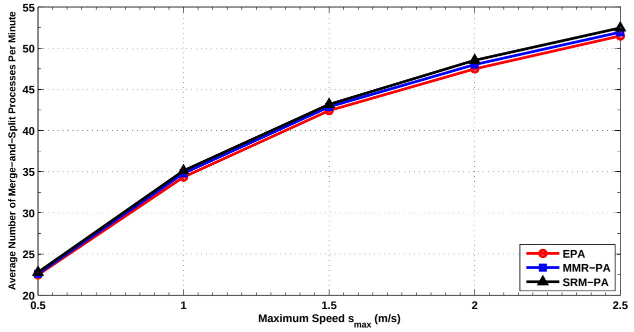

Fig. 7. Average Number of Merge-and-Split Processes Under the Propose Distributed Algorithm - Network Density = 0.005 Nodes/Unit Square Area

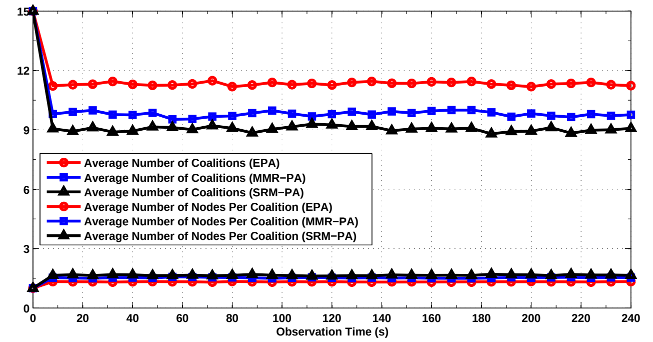

Fig. 8. Average Number of Coalitions and Number of Nodes Per Coalition Under Proposed Distributed Algorithm - Network Density = 0.005 Nodes/Unit Square Area and Maximum Speed $s_{\max} = 1.5$ m/s

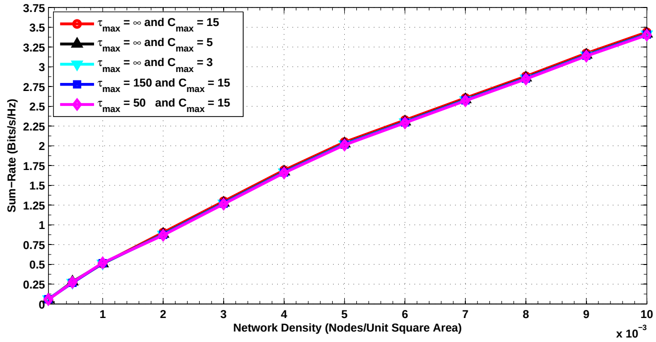

Fig. 9. Network Sum-Rate Under the Proposed Distributed Algorithm - EPA Criterion with Time-Index and Coalition Size Constraints

changes, which in turn triggers coalitions to either merge or split more often. Moreover, the SRM-PA criterion requires the highest average number of merge-and-split processes per minute. This is due to the fact that the SRM-PA criterion results in the highest number of coalitions among the different power allocation criteria and thus there is higher tendency to merge or split coalitions in response to network topology changes.

Fig. 8 illustrates the average number of coalitions and number of nodes per coalition as a function of time under the different power allocation criteria. As can be seen, the initial network structure starts with 15 singleton coalitions after which network nodes merge (or split) into larger (or smaller) coalitions. More importantly, the average number of coalitions and number of nodes per coalition agree with Fig. 4 (i.e. for the static network). This demonstrates that the proposed merge-and-split algorithm efficiently adapts to the nodes' mobility and topology changes.

Based on Fig. 5, the proposed merge-and-split algorithm converge—for instance—under the EPA criterion in about 170 iterations. To allow the proposed algorithm to converge faster and reduce the communication and computational complexities, especially under mobility, the algorithm time-index can be set to a maximum value of $\tau_{\max}$. An alternative method to speed up the convergence of the proposed algorithm is to restrict the maximum coalition size to $C_{\max}$. Fig. 9 shows that by reducing the value of $\tau_{\max}$ without restricting the coalition size (i.e. $C_{\max} = 15$), the degradation in the sum-rate is insignificant, even for $\tau_{\max} = 50$. Similarly, by reducing the maximum coalition size without capping the algorithm time-index (i.e. $\tau_{\max} = \infty$), the sum-rate marginally degrades. In Fig. 10, the average number of iterations for different combinations of time-index and coalition size restrictions is illustrated. Evidently, such constraints significantly reduces the number of iterations at the expense of negligible reduction in the network sum-rate.

Finally, Fig. 11 shows the percentage of nodes belonging to each coalition size under the different constraints. It is evident that decreasing the values of $\tau_{\max}$ and $C_{\max}$ prevents large coalitions from forming, which in turn increases the percentage of nodes remaining as singletons and decreases the percentage of nodes forming coalitions of larger sizes.

It is noteworthy that based on various network simulations, the performance of the proposed distributed merge-and-split algorithm has been found to usually lag that of the centralized algorithm, suggesting that there is no $\mathbb{D}_c$-stable partition in most cases (i.e. in these cases, the merge-and-split algorithm finds a suboptimal $\mathbb{D}_{hp}$-stable partition instead). In other words, in any network instance in which the centralized algorithm returns a better solution than the distributed merge-and-split algorithm, one can say for sure that there is not a $\mathbb{D}_c$-stable partition (because if there were a $\mathbb{D}_c$-stable partition, then it would have been optimal and the merge-and-split algorithm would have found it, so it would be impossible for the centralized algorithm to return a better solution). Therefore, our numerical results suggest that $\mathbb{D}_c$-stable partitions rarely exist in the simulated network instances.

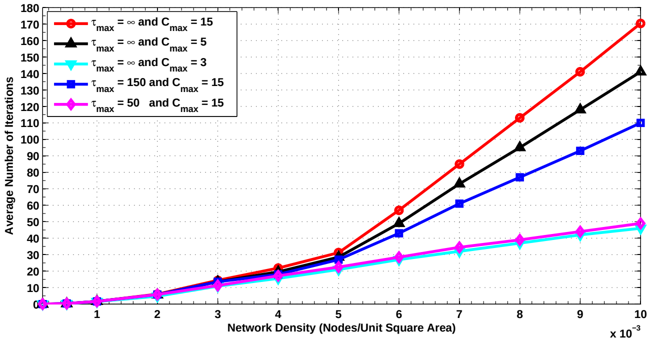

Fig. 10. Average Number of Iterations Under the Proposed Distributed Algorithm - EPA Criterion with Time-Index and Coalition Size Constraints

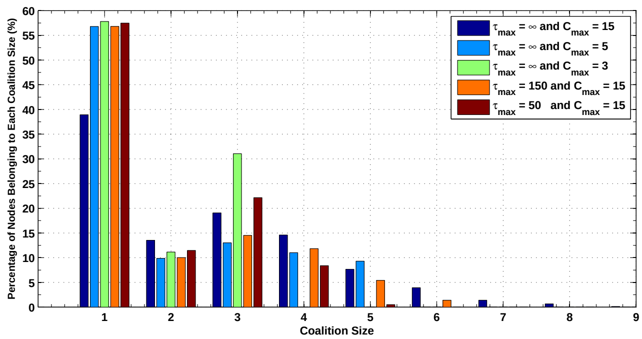

Fig. 11. Percentage of Nodes Belonging to Each Coalition Size Under the Proposed Distributed Algorithm - EPA Criterion with Time-Index and Coalition Size Constraints - Network Density = 0.01 Nodes/Unit Square Area

In [21, p. 157], an example ad-hoc network with $N = 10$ nodes and specific network topology is presented. Specifically, the initial and final network partitions under the centralized and distributed algorithms with different power allocation criteria are studied. Additionally, the resulting network sum-rates are summarized.

VIII. Discussion

A. Network Coding

As it is the case with common communication systems, our system model is based on TDMA, where each of the N network nodes is assigned a time-slot for transmission of its data. Additionally, there is a single time-slot that is allocated for multiple-access cooperative transmission. Therefore, the total number of time-slots is $N + 1$. The multiple-access cooperative transmission has been made possible via the use of network-coded transmission and multiuser detection at the destination node. In the case where the merge-and-split algorithm converges with no formed coalitions of two or more nodes, then the multiple-access time-slot will not be utilized and is thus wasted. In such case, the achievable network sum-rate is less than that of direct transmissions' sum-rate. This happens when the network nodes are far away from each other (i.e. network density is low). On the other hand, when the network nodes are within close proximity of each (i.e. network density is high), then coalitions of two or more nodes are more likely to be formed. In this case, the multiple-access time-slot is utilized and the achievable network sum-rate is greater than the sum of the direct transmission rates.

In conventional TDMA-based relay communications (without network-coding and multiuser detection), the total number of time-slots for a network of K coalitions (where $1 \leq K \leq N$) is determined as [1]

$$\mathcal{T} = N + \sum_{k=1}^{K} \mathbb{I}(C_k) \cdot |C_k|(|C_k| - 1), \qquad (23)$$

where $\mathbb{I}(C_k) = 1$ if $|C_k| \geq 2$, and 0 otherwise. In the case of singletons, $\mathcal{T} = N$. However, in the case of a grand coalition, then $\mathcal{T} = N^2$. For instance, when $K = N - 1$ (i.e. there is a coalition of size 2), then $\mathcal{T} = N+2$. Clearly, our system model is thus more bandwidth efficient than conventional cooperative communication systems.

B. Coalition-Head Selection

A coalition-head can be selected randomly by the nodes in the coalition (by exchanging an advertisement message). For instance, in the coalition formation phase, all the nodes in the coalition could select a coalition-head by selecting a random number between 0 and 1. The node with the smallest/largest number is then selected as coalition-head [24]. Coalition-head selection may also be based on a node's attributes such as their identification number, location or available transmission or processing resources [25] [26].

It should be noted that the coalition-head is responsible for periodically broadcasting timing information to the rest of the coalition and determining the power allocation fractions (which are convex optimization problems—see Section V). Such tasks do not pose severe computational overhead on the coalition-head. However, for fairness, the task of being a coalition-head can be performed in a round-robin fashion. Interested readers can also refer to [8,9], where message exchanges for coalition-head selection and coalition formation are discussed.

C. Timing Synchronization

In practice, for distributed timing synchronization, the coalition-head is responsible for exchanging timing information (i.e. the SYNC signal) through periodic beacon transmissions via appropriate control channels. The other nodes synchronize their clocks according to a time-stamp in the beacon [15] [16].

It should be noted that mismatches in clocks of the geographically distributed nodes results in mis-synchronized transmissions. However, due to the target SNR requirement within each coalition, only a small number of nodes within close proximity of each other form a coalition. Therefore, it is reasonable to assume that the nodes within a single coalition are perfectly synchronized with the coalition-head. In this case and from a network perspective, each coalition can be considered as a single entity and thus there is mis-synchronization between the coalitions and the destination node. However, this case is beyond the scope of this work as our focus has been to study distributed altruistic coalition formation and the impact of different power allocation criteria. Intuitively, it is expected that the achievable rate of each node under imperfect timing synchronization to degrade.

D. Mobility

In mobile ad-hoc wireless networks (MANETs), nodes are mobile and thus topology changes are inevitable. In such case, nodes move in and out of the communication range of other nodes, leading coalitions to be physically separated and communication links to fail. In [27], a cooperation model has been proposed based on coalitional game theory to incentivize cooperation, while trying to restore average node reachability upon topology changes caused by mobility. Particularly, the proposed model maintains and restructures the formed coalitions to restore reachability and stability. The future work in this direction aims at studying coalition formations for many-to-many cooperative communications while incorporating reachability under different individual and group mobility models.

E. Power Allocation

Although this paper has analyzed and compared the impact of different power allocation criteria on coalition formation, the selection of the power allocation criterion is application-specific. For instance, if the aim is to maximize the network sum-rate, then the SRM-PA criterion will be utilized throughout the network operation. Moreover, if the aim is to maximize the minimum rate of each node in the network so as to achieve rate-fairness, then the MMR-PA criterion is used. Finally, if the aim is to allocate power with minimal communication overhead and computational complexity and no specific rate requirement, then the EPA criterion is used.

IX. Conclusions

In this paper, altruistic coalition formation for cooperative relay networks has been studied. A distributed merge-and-split algorithm has been designed based on the utilitarian order and under different cooperative power allocation criteria. It has been shown that the proposed algorithm allows network nodes to self-organize into disjoint coalitions and that the sum-of-rates maximizing power allocation criterion results in the largest average coalition size and number of nodes per coalition, among the different power allocation criteria. Centralized power allocation and coalition formation are also investigated, where it shown that the proposed algorithm achieves a network sum-rate that is comparable with that of a centralized controller; however, with less computational complexity. Finally, the proposed algorithm has been shown to efficiently adapt to nodes' mobility and network topology changes.

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