Power Allocation Over Time-Varying Multi-User Multi-Relay Amplify-and-Forward Networks

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Abstract

In this paper, power allocation over time-varying multi-user multi-relay amplify-and-forward networks is studied. Specifically, stochastic network sum-rate, max-min rate power allocation and total power minimization problems are formulated. However, solving such stochastic problems relies on perfect global instantaneous channel state information (CSI), and thus entails complex computations and excessive communication overheads. To circumvent these issues, second-order statistics of the CSI (i.e. partial CSI) are utilized to transform the stochastic formulations into deterministic optimization problems in terms of ergodic capacity while satisfying quality-of-service constraints via target outage probability. The obtained optimal deterministic problems are non-convex and thus are computationally prohibitive. However, at high enough signal-to-noise ratio, such problems can be transformed into asymptotically convex ones, and thus solved efficiently. Simulation results illustrate that the proposed approximate deterministic power allocation reformulations closely agree with their optimal exact deterministic and dynamic counterparts.

Index Terms

Max-min, network sum-rate, outage probability, power allocation, relay channels, quality-of-service

I. Introduction

Cooperative relay networks have been proposed to mimic multiple-input multiple-output (MIMO) systems by forming virtual antenna arrays to exploit spatial diversity gains and improve network performance [1]. The benefits of such networks can be further reaped through optimal power allocation between the source and/or relay nodes, so as to improve different performance criteria. However, this requires knowledge of the channel state information at the relays and/or destination nodes. In time-varying wireless channels, optimal power allocation is a challenging task due to the need for accurate and complete global instantaneous channel state information (CSI). In turn, the power allocation task becomes a dynamic problem; requiring adaptive computationally-expensive algorithmic solutions, and excessive communication overheads. In practice, instantaneous CSI is obtained via channel estimation, which introduces estimation errors [2]. Consequently, such dynamic/adaptive algorithms experience significant performance loss, as the CSI may be erroneous and rather outdated, which has detrimental effects on the end-to-end signal-to-noise ratio (SNR), ultimately leading to sub-optimal performance and/or violation of quality-of-service (QoS) constraints [3,4]. Instead, each user should aim at maximizing its average achievable rate, while satisfying a probabilistic QoS specification (i.e. its average rate being not less than its target value [5]). To achieve this, it is imperative to consider alternative means to optimal power allocation with partial CSI, so as to achieve optimal/near-optimal performance while reducing computational complexity and communication overheads [6].

There has been a plethora of research works on power allocation in amplify-and-forward (AF) relay networks; however, most of them assume fixed channel gains or are based on dynamic algorithms with perfect knowledge of instantaneous CSI of all network users and/or relays [7]. For instance, in [8], the authors derive a closed-form optimal power allocation solution that requires instantaneous CSI for multiple AF relays, while incorporating total and individual power constraints. In [9], distributed and centralized power allocation for multiuser multi-relay AF networks is studied. Specifically, the authors consider the problems of maximization of the minimum rate, and the weighted sum-of-rates, which are based on perfect instantaneous CSI knowledge. In [10], a distributed iterative auction-based power allocation mechanism is proposed for deterministic multi-source multi-relay power allocation in relay networks, such that the network sum-rate is maximized. In particular, each source node must obtain complete CSI in order to be able to demand relay power based on the announced relay prices. In [11], the authors study the problem of minimizing the total transmission power subject to an outage constraint, and minimizing the outage probability subject to total transmit powers constraints in multi-hop multi-branch AF networks. To be specific, the authors derive asymptotically tight approximations of the received SNR, which are then used to formulate optimization problems using geometric programming (GP). Such GP problems are then transformed into nonlinear convex problems, which can be solved efficiently. In [12], the authors proposed efficient power allocation schemes for multi-source multi-relay AF networks, so as to maximize the network throughput and the minimum end-to-end SNR among the users, and also minimize the total transmit power of all users. Particularly, the proposed schemes are based on GP, which are transformed into equivalent convex optimization problems that can be solved efficiently. Total power minimization subject target symbol error rate (SER) QoS requirement is investigated in [13] for single-source multi-relay AF networks. Particularly, the authors derived closed-form power allocation solutions, and proposed power allocation algorithms to prolong network lifetime. Optimal power allocation for instantaneous SNR maximization in multi-hop AF networks under short-term (ST) and long-term (LT) power constraints is studied in [14]. Particularly, the authors illustrate that at sufficiently high SNR values, significant performance gains in terms of outage probability can be achieved with optimal power allocation over uniform power allocation under ST power constraint; while under the LT power constraint, substantial performance gain can be achieved in the low as well as high SNR regime. In [15], power allocation strategies for maximizing the end-to-end SNR, and minimizing the total power consumption while maintaining end-to-end SNR are studied for fixed-gain AF networks in Nakagami$-m$ fading channels. Specifically, the formulated strategies consider the all-participate as well as selective relaying under full and limited feedback, and are solved via convex optimization. The authors in [16] study the problem of energy efficiency (EE) for uplink transmission in AF orthogonal frequency division multiple-access (OFDMA) networks. Particularly, the aim is to assign subcarriers and allocate power to mobile and relay stations so as to maximize the EE of the mobile station (MS) with the lowest EE value. To that end, the authors formulated a primal max-min optimization problem subject to constraints on the MSs transmit power, relay station's transmit power and QoS of the MSs. Such problem turned out to be NP-hard as it involved non-convex fractional mixed integer nonlinear programming. In turn, a dual min-max optimization problem that attains the same optimal solution of the primal problem was provided, and a modified Dinkelback algorithm was proposed to obtain the optimal solution of the dual solution in polynomial-time complexity. Furthermore, a low-complexity suboptimal heuristic algorithm was also proposed to balance the achievable performance and computational complexity. In [17], the problem of joint subcarrier-relay assignment and power allocation for multi-user two-way multi-relay OFDMA networks is studied. Specifically, the problem of sum-rate maximization with individual power constraints is formulated as a mixed integer nonlinear programming problem, which is asymptotically solved via a Lagrange dual decomposition method. Moreover, a low-complexity suboptimal algorithm is proposed to trade off performance and complexity. The authors in [18] study the SER performance of M-ary phase shift keying and M-ary quadrature amplitude modulation of decode-and-forward (DF) networks over independent and non-identically distributed generalized fading channels. Particularly, exact analytic expressions involving the Lauricella function are derived and solved via a proposed computing algorithm. Moreover, asymptotic approximations at high SNR are obtained and optimum power allocation with total sum-power constraint is formulated and solved, yielding substantial performance enhancement over equal power allocation. Lastly, the problem of maximizing the minimum transmission rate among multiple source-destination pairs with multiple AF relays in a cognitive radio network is studied in [19]. Specifically, the joint relay assignment and channel allocation max-min rate problems for cognitive and cooperative communications (without and with network coding) are formulated, which are proved to be NP-hard. Consequently, reformulation and linearization techniques are applied and low-complexity algorithms are proposed to efficiently achieve high spectrum efficiency and improved max-min transmission rates. Generally speaking, the proposed optimal power allocation solutions/algorithms are based on instantaneous CSI, and this may not be suitable in time-varying channels [20]. Additionally, the computational complexity and communication overheads involved in dynamic power allocation may not be practical. Therefore, it is essential to consider power allocation strategies that do not rely on instantaneous CSI, and still be suitable for time-varying channels, while satisfying QoS constraints.

In this paper, power allocation over time-varying multi-user multi-relay amplify-and-forward networks is investigated. Particularly, the power allocation problems of network sum-rate (NSR) maximization, max-min rate (MMR) and total power minimization (TPM) are formulated as stochastic optimization problems, subject to QoS constraints (in terms of target rate or outage probability). The stochastic optimization problems are then transformed into optimal asymptotically convex deterministic problems at high enough SNR, where the time-varying rate function of each source-destination pair is replaced by its "time-average" ergodic rate function. Such transformations are based on the second-order channel statistics only (i.e. partial CSI)[1]. Consequently, they can be performed efficiently in an offline manner, as opposed to online "dynamic" power allocation, which requires complete instantaneous CSI. Our simulation results demonstrate that the convex approximations at high enough SNR closely agree with their derived optimal deterministic and dynamic counterparts. It should be noted that in [21], the authors study optimal power allocation for minimizing outage probability in the high SNR region subject to total power constraint, for the AF, DF and distributed space-time coding protocols. Particularly, convex approximations based on mean channel gain information are provided to achieve improvements in the outage probability while achieving significant coding gains. In [22], the authors derive an expression for the ergodic capacity and provide an upper-bound for a single-user multi-relay AF network. After that, they propose a novel quasi-optimal power allocation scheme that maximizes the upper-bound of the derived ergodic capacity and conclude that the cooperative mode should only be used when the source-to-destination channel gain is worse than that of the relay-to-destination. In [23], the performance of dual-hop MIMO relay

[1]The second-order channel statistics represent the mean channel gains (i.e. variances), and are assumed to encompass the effects of high data rates, mobility and carrier frequencies.

networks with statistical CSI is studied. Specifically, the authors derive the cumulative density function of the received SNR and closed-form expressions for the asymptotic fully correlated and uncorrelated scenarios. After that, a simple yet practical proportional power allocation algorithm is proposed, which is shown to agree with the benchmark performance obtained via exhaustive search. The problem of joint relay selection and power allocation in AF relay networks is studied in [24]. Particularly, the aim is to minimize the upper-bounded outage probability assuming only mean CSI is known. Then, the problem is decomposed into two parts, where a novel scheme is devised to incrementally select relays, and then optimally allocate power to the source and relays in the network. In [25], the authors study the achievable rate and ergodic capacity of non-orthogonal AF multi-relay networks with full CSI at the relays. Particularly, the authors determine the optimal instantaneous power amplification coefficients for achievable rate maximization, where the solution takes the form of an extended water-filling scheme. Similarly, it was shown that the ergodic capacity can be obtained via an iterative water-filling-based algorithm. After that, the authors study the capacity-achieving input covariance matrix in the high and low SNR regimes, where it has been shown that at sufficiently high SNR, the source transmit power must be equally distributed in all broadcasting phases; while in the low SNR regime, the source must spend all its power on the relay with the strongest source-relay and relay-destination channels.

To the best of our knowledge, no prior work has considered the formulation and transformation of stochastic network sum-rate maximizing and max-min rate power allocation, and total power minimization problems for multi-user multi-relay AF networks into their deterministic representations, and provide asymptotically convex approximations that are solvable with minimal computational complexity. More importantly, the formulated stochastic optimization problems capture the uncertainties, randomness and time-variations in the channel state information and incorporate them into the different power allocation optimization problems to achieve a more robust QoS performance and significantly improve network practicality.

The main contributions of this work are summarized as follow:

- Formulation of the network sum-rate and max-min rate power allocation, and total power minimization over time-varying multi-user multi-relay amplify-and-forward networks as stochastic optimization problems, and transforming them into optimal exact deterministic problems. Then, asymptotically convex approximations are provided, which are solved with minimal computational complexity.
- Comparing the convex approximate problems at high SNR with their optimal deterministic and dynamic counterparts, and demonstrating that they closely coincide with them.

It should be noted that a conference version of this paper has been accepted for publishing, which only studies the problems of network sum-rate maximization and max-min rate power allocation[2]. Particularly, the work in hand is an extension of the conference paper in the formulation of the total power minimization problem as a stochastic optimization problem, and transforming it into its optimal and approximate deterministic counterparts. Moreover, this paper provides additional discussion and comparative results demonstrating the applicability and potentials of the formulated asymptotically convex approximations under the different power allocation problems.

The remainder of this paper is organized as follows. Section II presents the stochastic and deterministic power allocation problems, while Section III provides the asymptotically convex reformulations. Section IV presents the stochastic total power minimization formulation as well as its deterministic convex approximation. Simulation results are presented in Section V, while conclusions are drawn in Section VI.

II. Power Allocation Formulations

A. Network Model

Consider an orthogonal "time-slotted" uplink cooperative relay network of N source-destination pairs and K AF relay nodes. Each source node S_i has transmit power of $P_{S_i}(t)$, for $i \in \{1, 2, \ldots, N\}$, while each relay R_k—for $k \in \{1, 2, \ldots, K\}$—allocates transmit power $P_{R_k,S_i}(t)$ to that source node. Let $h_{S_i,R_k}(t)$, $h_{R_k,D_i}(t)$ and $h_{S_i,D_i}(t)$ be the time-varying channel coefficients of the source-relay, relay-destination and source-destination links of nodes S_i, R_k and D_i, which are modeled as zero-mean complex Gaussian random variables with variances $\sigma^2_{S_i,R_k}$, $\sigma^2_{R_k,D_i}$ and $\sigma^2_{S_i,D_i}$, respectively. Additionally, each source-destination pair $S_i - D_i$ is assigned a signature waveform $c_i(t)$, which allows multiuser detection at the intended destination node [27,28]. Waveforms $c_i(t)$ and $c_j(t)$ have correlation coefficient $\rho_{i,j}$, where $0 \leq \rho_{i,j} \leq 1$ for $i \neq j$, and $\rho_{i,i} = 1$. For simplicity, let $\rho_{i,j} = \rho, \forall j \neq i$. It is assumed that there is a maximum power constraint $P_{\max}$ per time-slot t. Thus, $P_{S_i}(t) \leq P_{\max}$, $\forall i \in \{1, 2, \ldots, N\}$, and $\sum_{i=1}^{N} P_{R_k,S_i}(t) \leq P_{\max}$, $\forall k \in \{1, 2, \ldots, K\}$. Table I lists the main notations used in this paper.

Communication between each source-destination pair is performed over $N + K$ time-slots and is split into two phases, namely the broadcasting phase (of N time-slots), and the cooperation phase (of K time-slots) [1]. Particularly, each source node S_i is assigned a time-slot to broadcast its data symbol, which is received by each relay and destination node. After that, each relay R_k—in its assigned time-slot—forms a linearly-coded signal of all received signals and transmits it to the destination nodes, where

[2]A shorter "conference" version of this paper has recently been accepted for publication in the Proc. of the *IEEE International Wireless Communications and Mobile Computing (IWCMC) Conference, Paphos, Cyprus, Sept. 2016.* [26].

TABLE I

NOTATIONS

Symbol	Definition
D_i	Destination node i
$\mathbb{E}[\cdot]$	Expectation of parameter function
K	Number of amplify-and-forward relays in the network
M	Order of Laguerre polynomial
$\mathcal{M}_{(i)}(z)$	Moment generating function of the end-to-end SNR of source-destination pair $S_i - D_i$
$\mathcal{M}_{i,i}(z)$	Moment generating function of the direct transmission SNR of source-destination pair $S_i - D_i$
$\mathcal{M}_{k,i}(z)$	Moment generating function of the cooperative transmission SNR of source-destination pair $S_i - D_i$ via relay R_k
N	Number of source-destination pairs in the network
N_0	Noise variance
$\mathbb{P}[\cdot]$	Probability of parameter event
$P_{S_i}(t)$	Transmit power of source S_i during time-slot t
$P_{R_k,S_i}(t)$	Transmit power of relay R_k allocated to source S_i during time-slot t
$\mathbf{P}_{R,S_i}(t)$	Transmit relay power vector assigned by the K relays to source S_i during time-slot t
$P_{\max}$	Maximum transmit power constraint per time-slot
$\text{Pr}_{\text{Out},i}\left(\mathbf{P}_{R,S_i}\right)$	Outage probability as a function of $\mathbf{P}_{R,S_i}$
R_k	Relay node k
$R_i\left(\mathbf{P}_{R,S_i}(t)\right)$	Instantaneous achievable rate of source-destination pair $S_i - D_i$ as a function of $\mathbf{P}_{R,S_i}(t)$
R_T	Target rate
S_i	Source node i
α_m	Weight factor of the Laguerre polynomial
β_m	Abscissas of the Laguerre polynomial
$c_i(t)$	Signature waveform assigned to source S_i
$h_{S_i,D_i}(t)$	Time-varying Rayleigh fading channel coefficient between source S_i and destination D_i
$h_{S_i,R_k}(t)$	Time-varying Rayleigh fading channel coefficient between source S_i and relay R_k
$h_{R_k,D_i}(t)$	Time-varying Rayleigh fading channel coefficient between relay R_k and destination D_i
p_T	Target outage probability
ν	Path-loss exponent
$\sigma^2_{S_i,D_i}$	Channel variance between source S_i and destination D_i
$\sigma^2_{S_i,R_k}$	Channel variance between source S_i and relay R_k
$\sigma^2_{R_k,D_i}$	Channel variance between relay R_k and destination D_i
$\gamma_i(t)$	Instantaneous end-to-end SNR of source-destination pair $S_i - D_i$
$\gamma_{i,i}(t)$	Instantaneous SNR of direct transmission between source S_i and destination D_i
$\gamma_{k,i}(t)$	Instantaneous SNR of cooperative transmission of source-destination pair $S_i - D_i$ via relay R_k
ρ	Correlation coefficient of signature waveforms
ϱ	Noise amplification coefficient due to the use of signature waveforms

each destination node performs multiuser detection to separate the different users' data symbols [10,28]. The instantaneous SNR resulting at destination node D_i is given by [10,27]

$$\begin{aligned}\gamma_i(t) &= \gamma_{i,i}(t) + \sum_{k=1}^{K} \gamma_{k,i}(t) \\ &= \frac{P_{S_i}(t)|h_{S_i,D_i}(t)|^2}{N_0} + \sum_{k=1}^{K} \frac{1}{\varrho N_0} \frac{P_{S_i}(t)P_{R_k,S_i}(t)|h_{S_i,R_k}(t)|^2|h_{R_k,D_i}(t)|^2}{P_{S_i}(t)|h_{S_i,R_k}(t)|^2 + P_{R_k,S_i}(t)|h_{R_k,D_i}(t)|^2 + N_0},\end{aligned} \tag{1}$$

where $\gamma_{i,i}(t)$ is the instantaneous SNR for the direct transmission between source node S_i and destination node D_i (in the broadcasting phase); while $\gamma_{k,i}(t)$ refers to the instantaneous SNR of the cooperative transmission of source node S_i's signal to its intended destination D_i via relay R_k (in the cooperation phase). Moreover, N_0 is the noise variance, while ϱ is given by

$$\varrho = \frac{1+(N-2)\rho}{1+(N-2)\rho-(N-1)\rho^2}, \tag{2}$$

which represents the noise amplification coefficient resulting from the use of signature waveforms when performing multiuser detection at destination node D_i. It can be verified that when $\rho = 0$ (i.e. perfectly orthogonal signature waveforms) then $\varrho = 1$; otherwise $\varrho > 1$. Therefore, the higher the correlation is between the signature waveforms, the greater the noise amplification coefficient ϱ and the less the resulting end-to-end SNR. Lastly, one must note that $\gamma_{i,i}(t)$ is an exponential random variable with average rate $\lambda_{S_i,D_i} = \frac{N_0}{P_{S_i}\sigma^2_{S_i,D_i}}$.

Remark 1: Due to the strict increasing monotonicity of the SNR terms $\gamma_i(t)$ and $\gamma_{k,i}(t)$ in $P_{S_i}(t)$, $\forall i \in \{1,2,\ldots,N\}$ and $\forall k \in \{1,2,\ldots,K\}$, then $P_{S_i}(t) \triangleq P_S = P_{\max}$, $\forall i \in \{1,2,\ldots,N\}$, and $\forall t \geq 1$.

The achievable rate for each source-destination pair $S_i - D_i$ is given by [27]

$$R_i\left(\mathbf{P}_{R,S_i}(t)\right) = \frac{1}{N+K}\log_2\left(1+\gamma_{i,i}(t)+\sum_{k=1}^{K}\gamma_{k,i}(t)\right), \tag{3}$$

where $\mathbf{P}_{R,S_i}(t) = \left[P_{R_1,S_i}(t), P_{R_2,S_i}(t), \ldots, P_{R_K,S_i}(t)\right]^T$. Based on Remark 1, the rate function in (3) is a function of $P_{R_k,S_i}(t)$, $\forall k \in \{1,2,\ldots,K\}$ only.

B. Network Sum-Rate Maximization

The stochastic network sum-rate power allocation (S-NSR-PA) is formulated as

S-NSR-PA:

$$\begin{aligned} \max \quad & \textstyle\sum_{i=1}^{N} \mathbb{E}\left[R_i\left(\mathbf{P}_{R,S_i}(t)\right)\right] \\ \text{s.t.} \quad & \sum_{i=1}^{N} P_{R_k,S_i}(t) \leq P_{\max}, \qquad \forall k \in \{1,2,\ldots,K\}, && \text{(4a)} \\ & \mathbb{P}\left[R_i\left(\mathbf{P}_{R,S_i}(t)\right) \leq R_T\right] \leq p_T, \qquad \forall i \in \{1,2,\ldots,N\}, && \text{(4b)} \\ & P_{R_k,S_i}(t) \geq 0, \quad \forall k \in \{1,2,\ldots,K\} \text{ and } \forall i \in \{1,2,\ldots,N\}, && \text{(4c)} \end{aligned}$$

where R_T is the target rate; while p_T is the target outage probability. Moreover, $\mathbb{E}[\cdot]$ and $\mathbb{P}[\cdot]$ are the expectation and probability of the parameter function/event, respectively.

1) Average Network Sum-Rate: To simplify the analysis, the rate function in (3) can alternatively be expressed as [29]

$$R_i\left(\mathbf{P}_{R,S_i}(t)\right) = \frac{1}{(N+K)\ln 2}\int_0^{\infty}\frac{e^{-z}}{z}\left(1-e^{-z\left(\gamma_{i,i}(t)+\sum_{k=1}^{K}\gamma_{k,i}(t)\right)}\right)dz, \tag{5}$$

where the expectation of $R_i\left(\mathbf{P}_{R,S_i}(t)\right)$ is written as [29]

$$\mathbb{E}\left[R_i\left(\mathbf{P}_{R,S_i}(t)\right)\right] = \frac{1}{(N+K)\ln 2}\int_0^{\infty}\frac{e^{-z}}{z}\left(1-\mathcal{M}_{(i)}(z)\right)dz. \tag{6}$$

Additionally, $\mathcal{M}_{(i)}(z)$ is given by

$$\mathcal{M}_{(i)}(z) = \mathcal{M}_{i,i}(z)\cdot\prod_{k=1}^{K}\mathcal{M}_{k,i}(z), \tag{7}$$

where $\mathcal{M}_{i,i}(z)$ is given by [1]

$$\mathcal{M}_{i,i}(z) = \frac{1}{1+zP_S\sigma^2_{S_i,D_i}/N_0}. \tag{8}$$

while $\mathcal{M}_{k,i}(z)$ is given by [30]

$$\begin{aligned}\mathcal{M}_{k,i}(z) = &\frac{16\lambda_{S_i,R_k}\lambda_{R_k,D_i}}{3(\lambda_{S_i,R_k}+\lambda_{R_k,D_i}+2\sqrt{\lambda_{S_i,R_k}\lambda_{R_k,D_i}}+z)^2}\times\\ &\left[\frac{4(\lambda_{S_i,R_k}+\lambda_{R_k,D_i})}{\lambda_{S_i,R_k}+\lambda_{R_k,D_i}+2\sqrt{\lambda_{S_i,R_k}\lambda_{R_k,D_i}}+z}\times\ {}_2F_1\left(3,\frac{3}{2};\frac{5}{2};\frac{\lambda_{S_i,R_k}+\lambda_{R_k,D_i}-2\sqrt{\lambda_{S_i,R_k}\lambda_{R_k,D_i}}+z}{\lambda_{S_i,R_k}+\lambda_{R_k,D_i}+2\sqrt{\lambda_{S_i,R_k}\lambda_{R_k,D_i}}+z}\right)+\right.\\ &\left.{}_2F_1\left(2,\frac{1}{2};\frac{5}{2};\frac{\lambda_{S_i,R_k}+\lambda_{R_k,D_i}-2\sqrt{\lambda_{S_i,R_k}\lambda_{R_k,D_i}}+z}{\lambda_{S_i,R_k}+\lambda_{R_k,D_i}+2\sqrt{\lambda_{S_i,R_k}\lambda_{R_k,D_i}}+z}\right)\right],\end{aligned} \tag{9}$$

where ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ is the hypergeometric function [31], $\lambda_{S_i,R_k} = \frac{N_0\varrho}{P_S\sigma^2_{S_i,R_k}}$, and $\lambda_{R_k,D_i} = \frac{N_0\varrho}{P_{R_k,S_i}\sigma^2_{R_k,D_i}}$. Hence, the approximate average network sum-rate can be shown to be [32, Lemma 1]

$$\sum_{i=1}^{N}\mathbb{E}\left[R_i\left(\mathbf{P}_{R,S_i}\right)\right] = \frac{1}{(N+K)\ln 2}\cdot\sum_{m=1}^{M}\frac{\alpha_m}{\beta_m}\cdot\left[\sum_{i=1}^{N}\left[1-\mathcal{M}_{(i)}\left(\beta_m,\mathbf{P}_{R,S_i}\right)\right]\right], \tag{10}$$

where β_m and α_m are the abscissas and weight factors of the Laguerre polynomial, respectively (as given in [33, Table 25.9]), M is the order of the Laguerre polynomial, which defines the accuracy of the approximation.

2) Outage Probability: The approximate outage probability is expressed as [5]

$$\mathrm{Pr}_{\mathrm{Out},i}\left(\mathbf{P}_{R,S_i}\right) \simeq \sum_{k=1}^{K+1}\left(\prod_{m=1,m\neq k}^{K+1}\frac{\lambda_{S_i,R_m,D_i}}{\lambda_{S_i,R_m,D_i}-\lambda_{S_i,R_k,D_i}}\right)\left(1-e^{-\lambda_{S_i,R_k,D_i}\bar{R}_T}\right), \tag{11}$$

where $\bar{R}_T = 2^{(N+K)R_T}-1$, and

$$\lambda_{S_i,R_k,D_i} = \begin{cases}\lambda_{S_i,R_k}+\lambda_{R_k,D_i} = \varrho N_0\cdot\frac{P_S\sigma^2_{S_i,R_k}+P_{R_k,S_i}\sigma^2_{R_k,D_i}}{P_S P_{R_k,S_i}\sigma^2_{S_i,R_k}\sigma^2_{R_k,D_i}}, & \text{if } k\neq K+1\\ \lambda_{S_i,D_i} = \frac{N_0}{P_S\sigma^2_{S_i,D_i}}, & \text{if } k = K+1\end{cases}. \tag{12}$$

Therefore, the deterministic network sum-rate power allocation (D-NSR-PA) optimization problem is expressed as[3]

D-NSR-PA:

$$\begin{aligned}
\max \quad & \tfrac{1}{(N+K)\ln 2}\cdot\textstyle\sum_{m=1}^{M}\frac{\alpha_m}{\beta_m}\cdot\left[\sum_{i=1}^{N}\left[1-\mathcal{M}_{(i)}\left(\beta_m,\mathbf{P}_{R,S_i}\right)\right]\right]\\
\text{s.t.} \quad & \sum_{i=1}^{N}P_{R_k,S_i}\leq P_{\max}, \qquad \forall k\in\{1,2,\ldots,K\}, && \text{(13a)}\\
& \mathrm{Pr}_{\mathrm{Out},i}\left(\mathbf{P}_{R,S_i}\right)\leq p_T, \qquad \forall i\in\{1,2,\ldots,N\}, && \text{(13b)}\\
& P_{R_k,S_i}\geq 0, \quad \forall k\in\{1,2,\ldots,K\} \text{ and } \forall i\in\{1,2,\ldots,N\}. && \text{(13c)}
\end{aligned}$$

C. Max-Min Rate

The stochastic max-min rate power allocation (S-MMR-PA) problem is expressed as

S-MMR-PA:

$$\begin{aligned}
\max \quad & \min_{i\in\{1,2,\ldots,N\}}\mathbb{E}\left[R_i\left(\mathbf{P}_{R,S_i}(t)\right)\right]\\
\text{s.t.} \quad & \sum_{i=1}^{N}P_{R_k,S_i}(t)\leq P_{\max}, \quad \forall k\in\{1,2,\ldots,K\}, && \text{(14a)}\\
& \mathbb{P}\left[R_i\left(\mathbf{P}_{R,S_i}(t)\right)\leq R_T\right]\leq p_T, \qquad \forall i\in\{1,2,\ldots,N\}, && \text{(14b)}\\
& P_{R_k,S_i}(t)\geq 0, \quad \forall k\in\{1,2,\ldots,K\} \text{ and } \forall i\in\{1,2,\ldots,N\}. && \text{(14c)}
\end{aligned}$$

The above problem can be transformed into a deterministic max-min rate power allocation (D-MMR-PA) optimization problem as

[3]A deterministic optimization problem refers to one where the expectation of the time-varying rate function of each source-destination pair is replaced by its "time-average" ergodic rate function, and the outage probability is expressed in terms of the second-order channel statistics instead of the instantaneous channel conditions.

D-MMR-PA:

$$
\begin{aligned}
\max \quad & \eta \\
\text{s.t.} \quad & \sum_{i=1}^{N} P_{R_k,S_i} \leq P_{\max}, \quad \forall k \in \{1,2,\ldots,K\}, && \text{(15a)} \\
& \frac{1}{(N+K)\ln 2} \cdot \sum_{m=1}^{M} \frac{\alpha_m}{\beta_m} \cdot \left[1 - \mathcal{M}_{(i)}\left(\beta_m, \mathbf{P}_{R,S_i}\right)\right] \geq \eta, \quad \forall i \in \{1,2,\ldots,N\}, && \text{(15b)} \\
& \Pr_{\text{Out},i}\left(\mathbf{P}_{R,S_i}\right) \leq p_T, \quad \forall i \in \{1,2,\ldots,N\}, && \text{(15c)} \\
& P_{R_k,S_i} \geq 0, \qquad \forall k \in \{1,2,\ldots,K\} \text{ and } \forall i \in \{1,2,\ldots,N\}, && \text{(15d)} \\
& \eta \geq 0. && \text{(15e)}
\end{aligned}
$$

Remark 2: Problems **D-NSR-PA** and **D-MMR-PA** are non-convex, due to the non-convexity of the rate function and outage probability of each source-destination pair. Hence, they can only be solved efficiently using a global optimization software package.

III. Asymptotically Convex Power Allocation

This section provides approximate solutions to the network sum-rate maximization and max-min rate deterministic power allocation problems in the high SNR region.

The MGF function $\mathcal{M}_{(i)}(z)$ in (7)—at high enough SNR—can be approximated as [1]

$$\overline{\mathcal{M}}_{(i)}(z) = \overline{\mathcal{M}}_{i,i}(z) \cdot \prod_{k=1}^{K} \overline{\mathcal{M}}_{k,i}(z), \tag{16}$$

where

$$\overline{\mathcal{M}}_{i,i}(z) \simeq \frac{N_0}{z P_S \sigma^2_{S_i,D_i}}, \tag{17}$$

while

$$\overline{\mathcal{M}}_{k,i}(z) \simeq \frac{\varrho N_0}{z} \cdot \frac{P_S \sigma^2_{S_i,R_k} + P_{R_k,S_i}\sigma^2_{R_k,D_i}}{P_S P_{R_k,S_i} \sigma^2_{S_i,R_k} \sigma^2_{R_k,D_i}}. \tag{18}$$

Note that $\overline{\mathcal{M}}_{i,i}(z)$ in (17) is independent of P_{R_k,S_i}, $\forall k \in \{1,2,\ldots,K\}$.

The outage probability is tightly approximated and upper-bounded at high enough SNR as [5]

$$\overline{\Pr}_{\text{Out},i}\left(\mathbf{P}_{R,S_i}\right) \simeq \frac{\left(\bar{R}_T\right)^{K+1}}{(K+1)!} \cdot \frac{N_0}{P_S \sigma^2_{S_i,D_i}} \cdot \prod_{k=1}^{K} \varrho N_0 \frac{P_S \sigma^2_{S_i,R_k} + P_{R_k,S_i}\sigma^2_{R_k,D_i}}{P_S P_{R_k,S_i} \sigma^2_{S_i,R_k} \sigma^2_{R_k,D_i}}. \tag{19}$$

Remark 3: It can be verified that $\overline{\mathcal{M}}_{(i)}(z)$ and $\overline{\Pr}_{\text{Out},i}\left(\mathbf{P}_{R,S_i}\right)$ are convex in P_{R_k,S_i}, $\forall i \in \{1,2,\ldots,N\}$ and $\forall k \in \{1,2,\ldots,K\}$ [5].

Proof: See Appendix I.A.

At high enough SNR, the approximate deterministic network sum-rate power allocation (A-D-NSR-PA) problem is given by

A-D-NSR-PA:

$$\begin{aligned} \max \quad & \tfrac{1}{(N+K)\ln 2} \cdot \textstyle\sum_{m=1}^{M} \tfrac{\alpha_m}{\beta_m} \cdot \left[\textstyle\sum_{i=1}^{N} \left[1 - \overline{\mathcal{M}}_{(i)}\left(\beta_m, \mathbf{P}_{R,S_i}\right)\right]\right] \\ \text{s.t.} \quad & \sum_{i=1}^{N} P_{R_k,S_i} \leq P_{\max}, \qquad \forall k \in \{1,2,\ldots,K\}, && \text{(20a)} \\ & \overline{\Pr}_{\text{Out},i}\left(\mathbf{P}_{R,S_i}\right) \leq p_T, \qquad \forall i \in \{1,2,\ldots,N\}, && \text{(20b)} \\ & P_{R_k,S_i} \geq 0, \quad \forall k \in \{1,2,\ldots,K\} \text{ and } \forall i \in \{1,2,\ldots,N\}. && \text{(20c)} \end{aligned}$$

Remark 4: It can be easily verified that $-\overline{\mathcal{M}}_{(i)}\left(\beta_m, \mathbf{P}_{R,S_i}\right)$ is concave in $P_{R_k,S_i}, \forall k \in \{1,2,\ldots,K\}$.

Proof: See Appendix I.B.

A. Max-Min Rate

Similarly, the approximate deterministic max-min rate power allocation (A-D-MMR-PA) problem at high enough SNR is expressed as

A-D-MMR-PA:

$$\begin{aligned} \max \quad & \eta \\ \text{s.t.} \quad & \sum_{i=1}^{N} P_{R_k,S_i} \leq P_{\max}, \quad \forall k \in \{1,2,\ldots,K\}, && \text{(21a)} \\ & \frac{1}{(N+K)\ln 2} \cdot \sum_{m=1}^{M} \frac{\alpha_m}{\beta_m} \cdot \left[1 - \overline{\mathcal{M}}_{(i)}\left(\beta_m, \mathbf{P}_{R,S_i}\right)\right] \geq \eta, \quad \forall i \in \{1,2,\ldots,N\}, && \text{(21b)} \\ & \overline{\Pr}_{\text{Out},i}\left(\mathbf{P}_{R,S_i}\right) \leq p_T, \quad \forall i \in \{1,2,\ldots,N\}, && \text{(21c)} \\ & P_{R_k,S_i} \geq 0, \qquad \forall k \in \{1,2,\ldots,K\} \text{ and } \forall i \in \{1,2,\ldots,N\}, && \text{(21d)} \\ & \eta \geq 0. && \text{(21e)} \end{aligned}$$

Remark 5: Problems **A-D-NSR-PA** and **A-D-MMR-PA** are convex optimization problems and thus can be efficiently solved in polynomial-time complexity using any standard convex optimization software package with interior-point methods [34,35].

IV. Total Power Minimization

The stochastic total power minimization (S-TPM) problem is formulated as

S-TPM:

$$\begin{aligned}
\min \quad & \textstyle\sum_{k=1}^{K} \sum_{i=1}^{N} P_{R_k,S_i}(t) \\
\text{s.t.} \quad & \sum_{i=1}^{N} P_{R_k,S_i}(t) \leq P_{\max}, \qquad \forall k \in \{1,2,\ldots,K\}, && \text{(22a)} \\
& \mathbb{E}\left[R_i\left(\mathbf{P}_{R,S_i}(t)\right)\right] \geq R_T, \qquad \forall i \in \{1,2,\ldots,N\}, && \text{(22b)} \\
& P_{R_k,S_i}(t) \geq 0, \quad \forall k \in \{1,2,\ldots,K\} \text{ and } \forall i \in \{1,2,\ldots,N\}, && \text{(22c)}
\end{aligned}$$

which is transformed into a deterministic total power minimization (D-TPM) problem as

D-TPM:

$$\begin{aligned}
\min \quad & \textstyle\sum_{k=1}^{K} \sum_{i=1}^{N} P_{R_k,S_i} \\
\text{s.t.} \quad & \sum_{i=1}^{N} P_{R_k,S_i} \leq P_{\max}, \quad \forall k \in \{1,2,\ldots,K\}, && \text{(23a)} \\
& \frac{1}{(N+K)\ln 2} \cdot \sum_{m=1}^{M} \frac{\alpha_m}{\beta_m} \cdot \left[1 - \mathcal{M}_{(i)}\left(\beta_m, \mathbf{P}_{R,S_i}\right)\right] \geq R_T, \quad \forall i \in \{1,2,\ldots,N\}, && \text{(23b)} \\
& P_{R_k,S_i} \geq 0, \quad \forall k \in \{1,2,\ldots,K\} \text{ and } \forall i \in \{1,2,\ldots,N\}, && \text{(23c)}
\end{aligned}$$

which is non-convex due to the non-convexity of the rate function in (23b). However, by using (16), the approximate deterministic total power minimization (A-D-TPM) problem at high enough SNR is written as

A-D-TPM:

$$\begin{aligned}
\min \quad & \textstyle\sum_{k=1}^{K} \sum_{i=1}^{N} P_{R_k,S_i} \\
\text{s.t.} \quad & \sum_{i=1}^{N} P_{R_k,S_i} \leq P_{\max}, \quad \forall k \in \{1,2,\ldots,K\}, && \text{(24a)} \\
& \frac{1}{(N+K)\ln 2} \cdot \sum_{m=1}^{M} \frac{\alpha_m}{\beta_m} \cdot \left[1 - \overline{\mathcal{M}}_{(i)}\left(\beta_m, \mathbf{P}_{R,S_i}\right)\right] \geq R_T, \quad \forall i \in \{1,2,\ldots,N\}, && \text{(24b)} \\
& P_{R_k,S_i} \geq 0, \quad \forall k \in \{1,2,\ldots,K\} \text{ and } \forall i \in \{1,2,\ldots,N\}, && \text{(24c)}
\end{aligned}$$

which is now a convex optimization problem.

V. Simulation Results

This section evaluates the formulated optimal deterministic power allocation problems and compares them with the approximated problems and equal power allocation (EPA)[4]. Moreover, dynamic optimal power allocation (Dyn-OPA) is also compared and formulated as

Dyn-OPA:

$$\max \quad f\left(R_1\left(\mathbf{P}_{R,S_1}(t)\right), R_2\left(\mathbf{P}_{R,S_2}(t)\right), \ldots, R_N\left(\mathbf{P}_{R,S_N}(t)\right)\right)$$

$$\text{s.t.} \quad \sum_{i=1}^{N} P_{R_k,S_i}(t) \leq P_{\max}, \qquad \forall k \in \{1,2,\ldots,K\}, \tag{25a}$$

$$R_i\left(\mathbf{P}_{R,S_i}(t)\right) \geq R_T, \qquad \forall i \in \{1,2,\ldots,N\}, \tag{25b}$$

$$P_{R_k,S_i}(t) \geq 0, \quad \forall k \in \{1,2,\ldots,K\} \text{ and } \forall i \in \{1,2,\ldots,N\}. \tag{25c}$$

where

$$f\left(R_1\left(\mathbf{P}_{R,S_1}(t)\right), R_2\left(\mathbf{P}_{R,S_2}(t)\right), \ldots, R_N\left(\mathbf{P}_{R,S_N}(t)\right)\right) = \begin{cases} \sum_{i=1}^{N} R_i\left(\mathbf{P}_{R,S_i}(t)\right), & \text{for NSR} \\ \min_{i\in\{1,2,\ldots,N\}} R_i\left(\mathbf{P}_{R,S_i}(t)\right), & \text{for MMR.} \end{cases} \tag{26}$$

Additionally, the dynamic optimal total power minimization (Dyn-O-TPM) problem is expressed as[5]

Dyn-O-TPM:

$$\min \quad \sum_{k=1}^{K} \sum_{i=1}^{N} P_{R_k,S_i}(t)$$

$$\text{s.t.} \quad \sum_{i=1}^{N} P_{R_k,S_i}(t) \leq P_{\max}, \qquad \forall k \in \{1,2,\ldots,K\}, \tag{27a}$$

$$R_i\left(\mathbf{P}_{R,S_i}(t)\right) \geq R_T, \qquad \forall i \in \{1,2,\ldots,N\}, \tag{27b}$$

$$P_{R_k,S_i}(t) \geq 0, \quad \forall k \in \{1,2,\ldots,K\} \text{ and } \forall i \in \{1,2,\ldots,N\}. \tag{27c}$$

The network topology over an area of 2.5m $\times$ 3.5m is illustrated in Fig. 1, which consists of $N = 3$ source-destination pairs and $K = 2$ relay nodes. Moreover, the channel gain between any two nodes is given by $\sigma^2 = d^{-\nu}$, where d and ν are the inter-node distance and path-loss exponent, respectively.

[4]EPA implies that in each time-slot of the cooperation phase, the total power constraint $P_{\max}$ of each relay R_k is equally split across the N source nodes. That is, $P_{R_k,S_i} = P_{\max}/N$, $\forall i \in \{1,2,\ldots,N\}$ and $\forall k \in \{1,2,\ldots,K\}$.

[5]The deterministic optimal, approximate and dynamic optimization problems are solved via MIDACO, with tolerance set to 0.0001 [37].

Moreover, the simulations are averaged over 10^6 independent runs with randomly generated channel coefficients that change every time-slot[6]. The simulation parameters are summarized in Table II.

TABLE II
SIMULATION PARAMETERS

Parameter	$P_{\max}$	ν	ρ	R_T
Value	150 mW	2.5	0.15	0.5 Bits/s/Hz

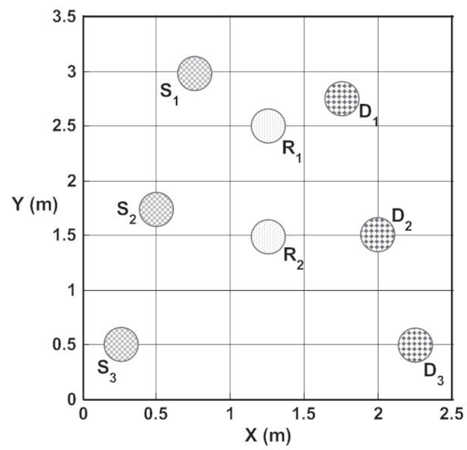

Fig. 1. Network Topology

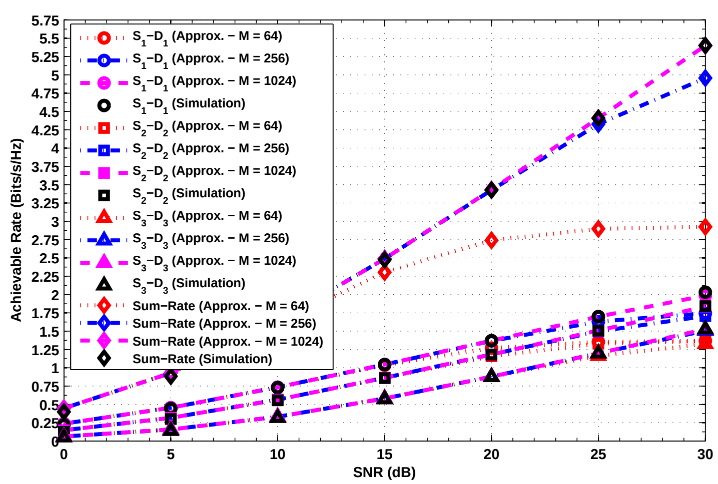

Fig. 2. Achievable Rate: Approximation vs. Simulation - EPA

[6]Dynamic optimal power allocation and total power minimization are performed at the end of the broadcasting phase (i.e. before the cooperation phase), and assumed to be achieved via a centralized controller with perfect CSI.

Fig. 2 demonstrates the average rate per source-destination pair as well as the average network sum-rate under relay equal power allocation (EPA) and for different orders of approximation M. One can see that source-destination pair $S_1 - D_1$ achieves the highest rate, when compared with the $S_2 - D_2$ and $S_3 - D_3$ pairs. This is explained by noting that nodes S_1 and D_1 are relatively closer to each other than the nodes of the other two pairs. Additionally, nodes S_1 and D_1 are relatively closer to relays R_1 and R_2 than the other source/destination nodes, which implies less path-loss and channel noise. This also explains why the pair $S_3 - D_3$ achieves the lowest average rate. It is also clear that increasing the order of approximation M improves the accuracy of the average rate of each source-destination pair, in comparison with the simulated rates. For example, $M = 256$ suffices for accurate average rate approximation at SNR = 20 dB; while $M = 1024$ is sufficient at SNR = 30 dB (i.e. higher values of SNR require greater values of M).

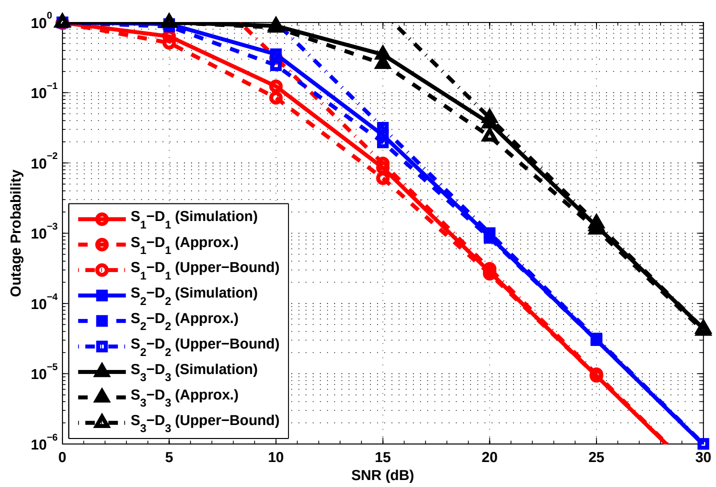

Fig. 3. Outage Probability - EPA

Fig. 3 illustrates the simulated outage probability per source-destination pair under EPA, in comparison with the theoretical approximate and upper-bound outage probabilities, as given by (13) and (21), respectively. Clearly, the theoretical approximate and upper-bound outage probabilities agree with their simulated counterpart at SNR = 30 dB, for all source-destination pairs. Moreover, the pair $S_3 - D_3$ achieves the highest outage probability of 4×10^{-5}, which agrees with the observation that this pair achieves the lowest average rate.

In Fig. 4, the average network sum-rate of the different power allocation problems are compared at SNR = 30 dB, when $p_T = 10^{-5}$ and $M = 1024$. It is clear that the convex approximate problems **A-D-MMR-PA** and **A-D-NSR-PA** agree with their optimal counterparts **D-MMR-PA** and **D-NSR-PA**

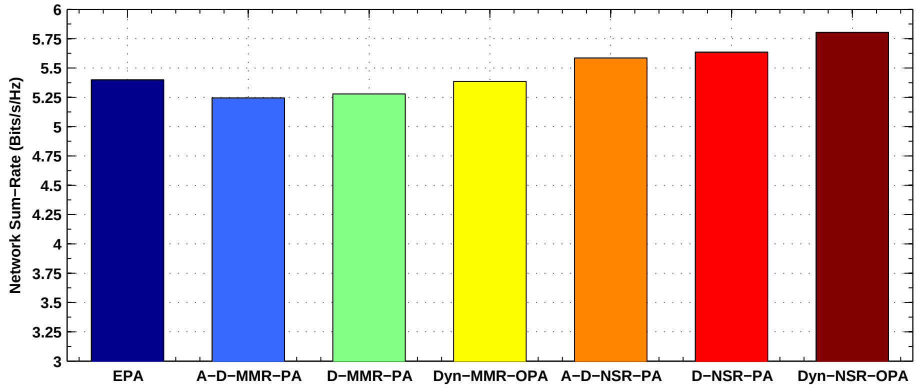

Fig. 4. Comparison of Network Sum-Rate - $p_T = 10^{-5}$, SNR = 30 dB and $M = 1024$

to within 0.03 bits/s/Hz. Moreover, the average network sum-rate under the NSR-PA problems is higher than their MMR-PA counterparts. Additionally, it is evident that the network sum-rates resulting from the **Dyn-MMR-OPA** and **Dyn-NSR-OPA** problems are superior to their deterministic counterparts, and this is due to the fact that they relay on complete instantaneous CSI, rather than partial CSI. Lastly, one can see that EPA is superior to the deterministic MMR-PA problems and yields almost the same network sum-rate as the dynamic MMR-PA problem. This is because the MMR-PA problems tend to make the rates of the different source nodes equal, which poses a tradeoff between the network sum-rate and fairness.

Fig 5a presents the average rate per source-destination pair under the different power allocation problems, where it can be seen that the rates are almost equal under the MMR-PA problems, as expected. On the other hand, the pair $S_1 - D_1$ achieves the highest average rate under the NSR-PA problems, as noted before. Moreover, the dynamic power allocation problems yield higher rate per source-destination pair than their deterministic counterparts. Fig. 5b illustrates the outage probability per source-destination pair, where it is evident that all the power allocation problems closely satisfy the target outage probability $p_T \lesssim 10^{-5}$ for all source-destination pairs, except for the pair $S_3 - D_3$ under the EPA (as noted in Fig. 3). Finally, the NSR-PA problems achieve the lowest outage probability while satisfying the target outage probability p_T for all pairs, when compared with the other problems.

Fig. 6 demonstrates the minimum total power required to satisfy a target rate R_T under the exact deterministic total power minimizing problem **D-TPM**, its asymptotically convex counterpart **A-D-TPM**, and dynamic optimal total power minimizing problem **Dyn-O-TPM**. Clearly, one can see that for the deterministic problems for target rates $R_T = 1.5$ and $R_T = 1.6$ bits/s/Hz, the minimum total power is almost equal, with negligible difference. Additionally, it can be seen that the minimum total power

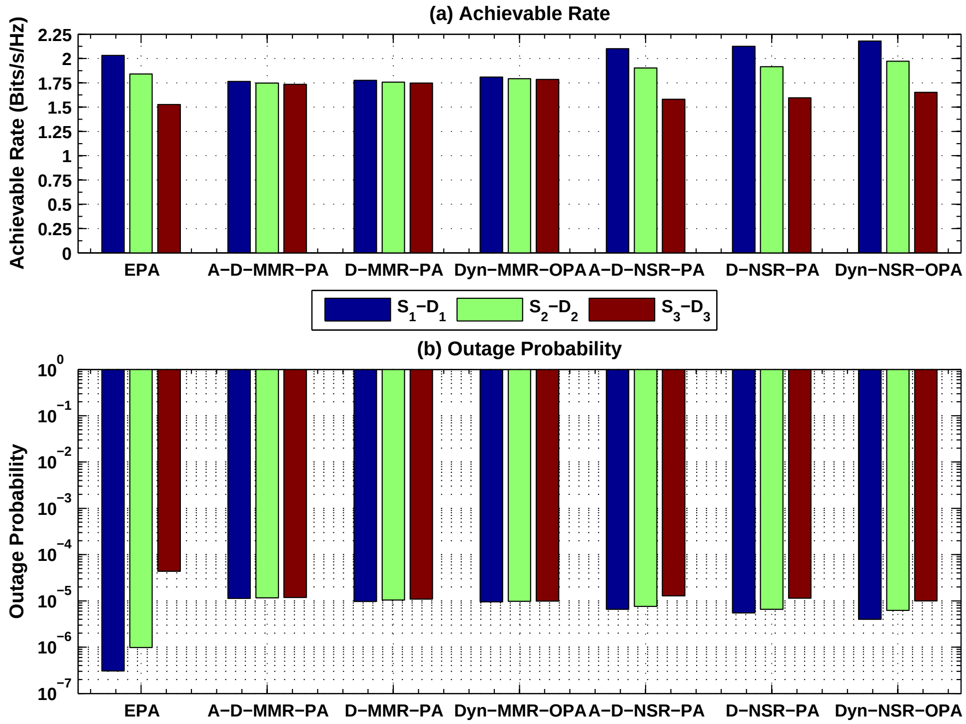

Fig. 5. Comparison of (a) Achievable Rate, and (b) Outage Probability of Each Source-Destination Pair - $p_T = 10^{-5}$, SNR = 30 dB and $M = 1024$

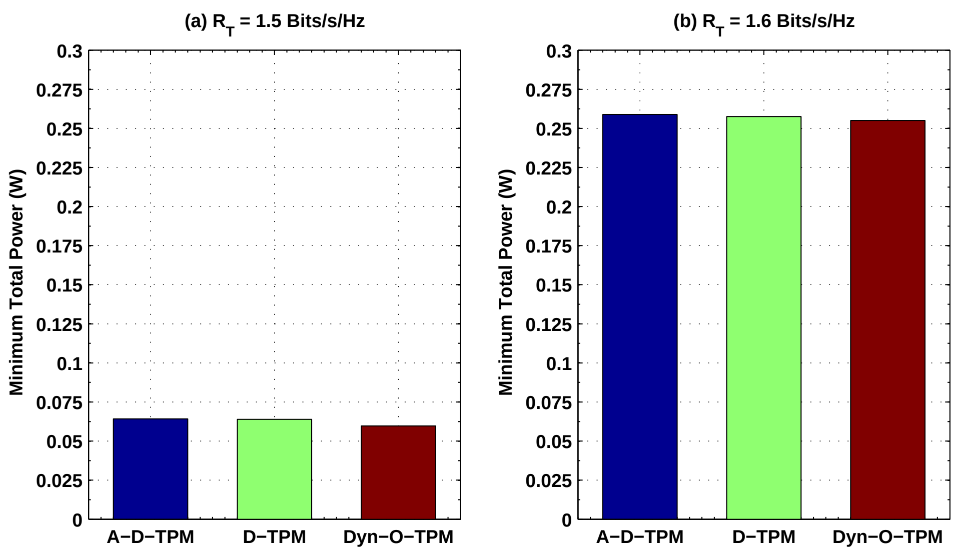

Fig. 6. Minimum Total Power for (a) $R_T = 1.5$ Bits/s/Hz, and (b) $R_T = 1.6$ Bits/s/Hz - SNR = 30 dB and $M = 1024$

required to satisfy the target rate of $R_T = 1.5$ bits/s/Hz (see Fig. 6a) is less than that when $R_T = 1.6$ bits/s/Hz (see Fig. 6b) under the different problems. This is because the lower the target rate R_T is, the smaller the amount of power required to satisfy it. Finally, the **Dyn-O-TPM** achieves the lowest minimum total power among the different problems and under both target rates. Again, this is attributed to the utilization of complete CSI for total power minimization.

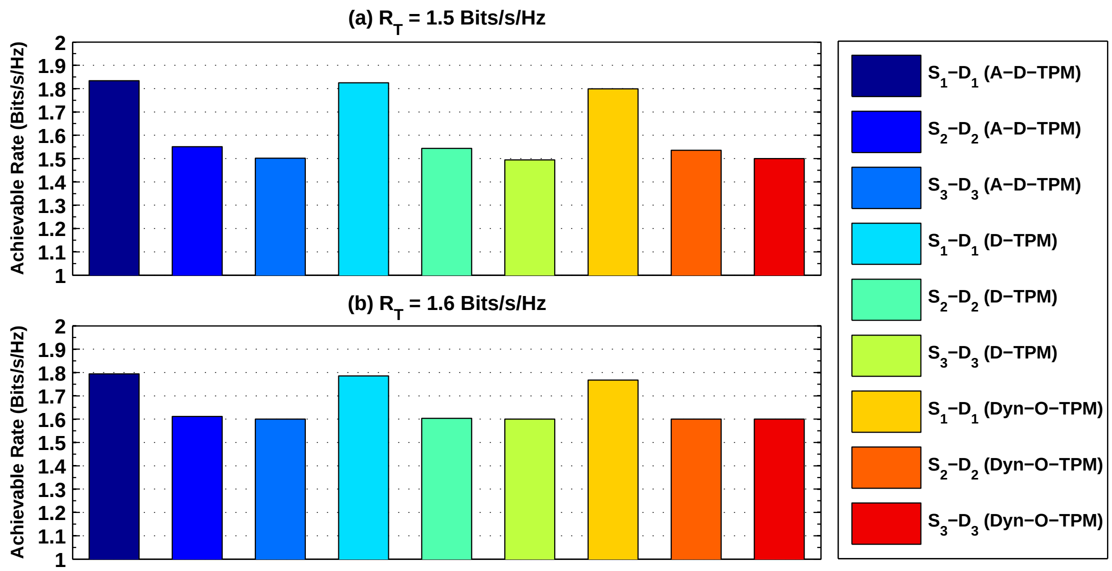

Fig. 7. Achievable Rate Per Source-Destination Pair for (a) $R_T = 1.5$ Bits/s/Hz, and (b) $R_T = 1.6$ Bits/s/Hz - SNR = 30 dB and $M = 1024$

In Fig. 7, the achievable rates per source-destination pair for different target rates under problems **D-TPM**, **A-D-TPM** and **Dyn-O-TPM** are shown. Specifically, in Fig. 7a (Fig. 7b), one can see that all source-destination pairs satisfy the target rate $R_T = 1.5$ bits/s/Hz ($R_T = 1.6$ bits/s/Hz). Additionally, one can see that the achievable rate of the pair $S_1 - D_1$ when $R_T = 1.5$ bit/s/Hz is higher than that when $R_T = 1.6$ bits/s/Hz. This is attributed to the fact that some of the transmit power allocated to the pair $S_1 - D_1$ when $R_T = 1.5$ bits/s/Hz is re-allocated to pairs $S_2 - D_2$ and $S_3 - D_3$ in order for them to satisfy the higher target rate of $R_T = 1.6$ bits/s/Hz, which in turn results in a dip to the rate of the $S_1 - D_1$ pair. Also, one can see that the resulting rates of each source-destination pair under the **D-TPM** problem is only marginally less than those of the **A-D-TPM** problem, since the total power under the former problem is marginally less than the latter problem (see Fig. 6). However, all source-destination pairs satisfy the target rates. This negligible difference is due to the asymptotic approximation. Finally, the rate per source-destination pair under the **Dyn-O-TPM** problem is less than their deterministic counterparts, since it requires the least amount of power to satisfy the target rates while relaying on complete instantaneous CSI.

In order to quantitatively demonstrate the applicability of the formulated convex approximate power allocation problems, three new comparison metrics are defined; the average network sum-rate efficiency (A-NSR-E), the average minimum total power efficiency (A-MTP-E), and the average achievable rate efficiency (A-AR-E). Particularly, the A-NSR-E, A-TMP-E, and A-AR-E metrics are expressed as

$$\text{A-NSR-E} = \frac{\text{Average Network Sum-Rate of Approximate Deterministic Power Allocation}}{\text{Average Network Sum-Rate of Dynamic Optimal Power Allocation}} \times 100, \tag{28}$$

$$\text{A-MTP-E} = \frac{\text{Average Minimum Total Power of Dynamic Optimal Power Allocation}}{\text{Average Minimum Total Power of Approximate Deterministic Power Allocation}} \times 100, \tag{29}$$

and

$$\text{A-AR-E} = \frac{\text{Average Achievable Rate of Dynamic Optimal Power Allocation}}{\text{Average Achievable Rate of Approximate Deterministic Power Allocation}} \times 100. \tag{30}$$

Fig. 8 illustrates the average network sum-rate efficiency of the **A-D-MMR-PA** and **A-D-NSR-PA** problems (derived from Fig. 4) when compared with the **Dyn-MMR-OPA** and **Dyn-NSR-OPA** problems, respectively. It is clear that the approximate deterministic power allocation problems achieve efficiencies not less than 96%, which proves that the convex approximate solutions closely coincide with their dynamic counterparts.

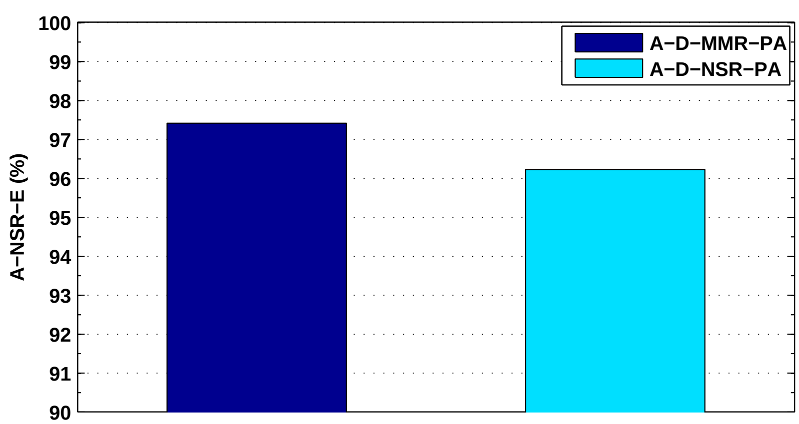

Fig. 8. Average Network Sum-Rate Efficiency - $p_T = 10^{-5}$, SNR = 30 dB and $M = 1024$

The average minimum total power efficiency of the approximate deterministic total power minimizing problem **A-D-TPM** relative to the dynamic optimal total power minimizing problem **Dyn-O-TPM** (as per Fig. 6) is illustrated in Fig. 9. Clearly, the A-MTP-E is 93% and 98.5% for $R_T = 1.5$ and $R_T = 1.6$ bits/s/Hz, respectively. Therefore, it is evident that the approximate deterministic problem agrees with its dynamic counterpart. Finally, in Fig. 10, the average achievable rate efficiency per source-destination under the **A-D-TPM** problem (as per Fig. 7) in comparison with the **Dyn-O-TPM** problem is demonstrated. Particularly, Fig. 10a illustrates the A-AR-E for each source-destination pair when $R_T = 1.5$ bits/s/Hz, where it can be seen that no pair achieves an efficiency less than 98%. A similar observation is noted in Fig. 10b for $R_T = 1.6$ bits/s/Hz, where it is evident that no pair achieves a relative efficiency less than 98.5% when compared with the dynamic problem. Hence, the afore-presented results demonstrate that the approximate deterministic power allocation problems provide average network sum-rate, minimum total power and achievable rate results that are comparable to those obtained via their dynamic counterparts.

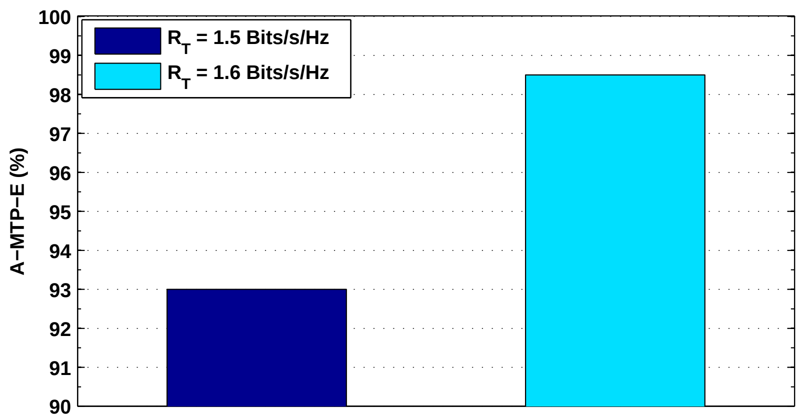

Fig. 9. Average Minimum Total Power Efficiency - SNR = 30 dB and $M = 1024$

The presented results demonstrate that the approximated average rate function of each source-destination pair is highly dependent on the order of approximation M. In fact, M must be large enough to yield faithful representation of the average rate of each source-destination pair. Moreover, it is intuitive that the higher the value of M is, the more computationally-intensive the solution of the approximate convex optimization problems. Additionally, it must be noted that using smaller values of M would yield solutions with less computational-complexity; however, such solutions would be far from optimal, in which case using equal power allocation would be more attractive and reasonable. It is also arguable that large values of M may entail complex computations and introduce significant delays. However, recently, primal-dual interior-point (and several other) methods have been proved to be extremely efficient in handling nonlinear large-scale convex problems, with polynomial-time complexity results obtained via reliable and robust software implementations (readily available in commercial and noncommercial packages) [35]. Particularly, such methods are able to solve dense problems of thousands of variables and over ten thousand terms almost as fast as linear programming problems [36]. Therefore, the derived

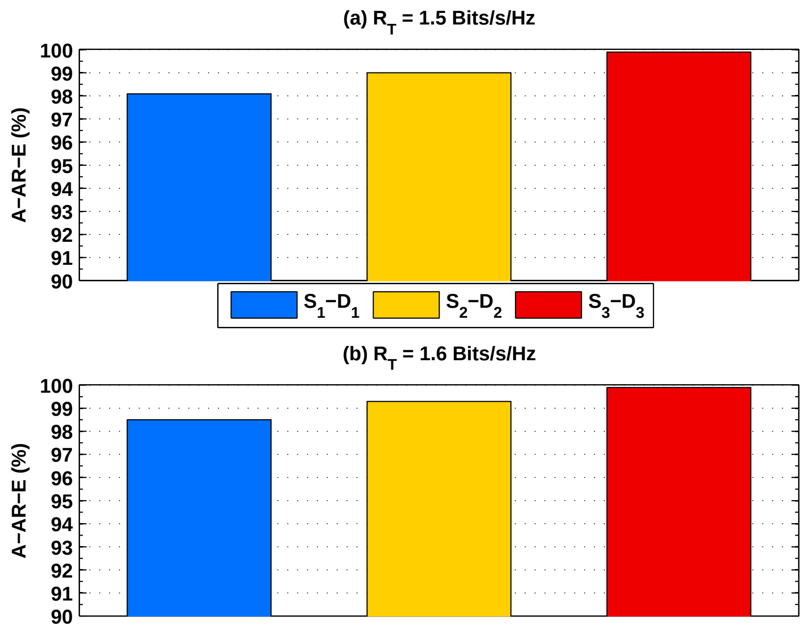

Fig. 10. Average Achievable Rate Efficiency Per Source-Destination Pair for (a) $R_T = 1.5$ Bits/s/Hz, and (b) $R_T = 1.6$ Bits/s/Hz - SNR = 30 dB and $M = 1024$

convex approximate deterministic power allocation problems can be efficiently solved in polynomial-time complexity via recent software optimization packages (as per Remark 5). Finally, it is noteworthy that the optimal deterministic power allocation problems took much longer to converge than the convex approximate ones. One must also keep in mind that these approximate convex problems are only solved once and in an offline manner, in the network set-up phase. Thus, they can be solved with minimal computation complexity, without introducing unnecessary communication overheads. On the other hand, the dynamic optimal power allocation problems must be re-executed every time the channel coefficients change. Although such problems can be shown to be convex (as verified in [9]), solving them under time-varying channel conditions is both computationally-expensive and communication-intensive, as they constantly require perfect global instantaneous CSI. Hence, the convex approximate power allocation problems are particularly more attractive in the case of mobility and rapidly changing channel conditions.

VI. Conclusions

In this paper, power allocation for time-varying multi-user multi-relay amplify-and-forward networks is studied. Specifically, stochastic optimization for network sum-rate maximization, max-min rate and total power minimization problems—subject to QoS constraints—have been formulated and transformed into deterministic asymptotically convex problems at high enough SNR. Our simulation results illustrate that the convex approximated problems closely agree with their optimal deterministic and dynamic counterparts. Specifically, the deterministic approximate problems have been shown to yield network sum-rates that are comparable with the deterministic as well as dynamic problems, while meeting the target outage probability per source-destination pair. Moreover, although the dynamic problems yield the minimum total relay transmit power when compared with their deterministic exact and approximate counterparts, the difference is negligibly small. Finally, if the communication and computational overheads associated with obtaining complete instantaneous CSI are factored into the performance of the dynamic power allocation problems, the difference in performance—when compared with the approximate convex problems—would be even less.

VII. Appendix I

A. *Proof of Convexity*

It should be noted that $\overline{\mathcal{M}}_{i,i}(z)$ in (17) is independent of P_{R_k,S_i}, $\forall k \in \{1, 2, \ldots, K\}$. Therefore, to prove the convexity of each MGF function $\overline{\mathcal{M}}_{k,i}(z)$, $\forall k \in \{1, 2, \ldots, K\}$, one must show that the second derivative is greater than zero $\forall P_{R_k,S_i} > 0$. Specifically, it is straightforward to show that

$$\frac{\partial \overline{\mathcal{M}}_{k,i}(z)}{\partial P_{R_k,S_i}} = -\frac{\varrho N_0}{z} \cdot \frac{1}{\sigma^2_{R_k,D_i} P^2_{R_k,S_i}} < 0, \tag{31}$$

and

$$\frac{\partial^2 \overline{\mathcal{M}}_{k,i}(z)}{\partial P^2_{R_k,S_i}} = \frac{\varrho N_0}{z} \cdot \frac{2}{\sigma^2_{R_k,D_i} P^3_{R_k,S_i}} > 0. \tag{32}$$

Clearly, the MGF $\overline{\mathcal{M}}_{k,i}(z)$ is a decreasing function (as the first derivative is negative) and strictly convex (since the second derivative is strictly positive $\forall P_{R_k,S_i} > 0$).

Now, in order to show that $\overline{\mathcal{M}}_{(i)}(z)$ is (strictly) convex in P_{R_k,S_i}, $\forall k \in \{1, 2, \ldots, K\}$, the *Hessian* matrix of all the second derivatives must be examined. For simplicity and mathematical traceability, assume there are only $K = 2$ relays, namely R_k and R_l for $k \neq l$. Hence, the Hessian matrix is written as

$$\begin{aligned} \mathbb{H}\left(P_{R_k,S_i}, P_{R_l,S_i}\right) &= \begin{bmatrix} \frac{\partial^2 \overline{\mathcal{M}}_{(i)}(z)}{\partial P^2_{R_k,S_i}} & \frac{\partial^2 \overline{\mathcal{M}}_{(i)}(z)}{\partial P_{R_k,S_i} \partial P_{R_l,S_i}} \\ \frac{\partial^2 \overline{\mathcal{M}}_{(i)}(z)}{\partial P_{R_l,S_i} \partial P_{R_k,S_i}} & \frac{\partial^2 \overline{\mathcal{M}}_{(i)}(z)}{\partial P^2_{R_l,S_i}} \end{bmatrix} \\ &= \overline{\mathcal{M}}_{i,i}(z) \cdot \left(\frac{\varrho N_0}{z}\right)^2 \cdot \begin{bmatrix} \frac{P_S \sigma^2_{S_i,R_l} + P_{R_l,S_i} \sigma^2_{R_l,D_i}}{P_S P_{R_l,S_i} \sigma^2_{S_i,R_l} \sigma^2_{R_l,D_i}} \cdot \frac{2}{\sigma^2_{R_k,D_i} P^3_{R_k,S_i}} & \frac{1}{P^2_{R_k,S_i} P^2_{R_l,S_i} \sigma^2_{R_k,D_i} \sigma^2_{R_l,D_i}} \\ \frac{1}{P^2_{R_k,S_i} P^2_{R_l,S_i} \sigma^2_{R_k,D_i} \sigma^2_{R_l,D_i}} & \frac{P_S \sigma^2_{S_i,R_k} + P_{R_k,S_i} \sigma^2_{R_k,D_i}}{P_S P_{R_k,S_i} \sigma^2_{S_i,R_k} \sigma^2_{R_k,D_i}} \cdot \frac{2}{\sigma^2_{R_l,D_i} P^3_{R_l,S_i}} \end{bmatrix}. \end{aligned} \tag{33}$$

The MGF function $\mathcal{M}_{(i)}(z)$ is (strictly) convex if and only if $\mathbb{H}\left(P_{R_k,S_i}, P_{R_l,S_i}\right)$ is positive (definite) semidefinite [34]. It is well-known (By Young's theorem [38]) that the Hessian of any function for which all second partial derivatives are continuous is symmetric for all values of the argument of the function. Also, all the diagonal elements are positive (since $\frac{\partial^2 \overline{\mathcal{M}}_{(i)}(z)}{\partial P^2_{R_k,S_i}} > 0$ and $\frac{\partial^2 \overline{\mathcal{M}}_{(i)}(z)}{\partial P^2_{R_l,S_i}} > 0$). Also, it can be verified that $\frac{\partial^2 \overline{\mathcal{M}}_{(i)}(z)}{\partial P^2_{R_k,S_i}} \frac{\partial^2 \overline{\mathcal{M}}_{(i)}(z)}{\partial P^2_{R_l,S_i}} - \left(\frac{\partial^2 \overline{\mathcal{M}}_{(i)}(z)}{\partial P_{R_l,S_i} \partial P_{R_k,S_i}}\right)^2 > 0$. Hence, $\mathcal{M}_{(i)}(z)$ is strictly convex in P_{R_k,S_i}, and P_{R_l,S_i}. In general, $\mathcal{M}_{(i)}(z)$ can be proved to (strictly) convex by verifying that it is positive definite in P_{R_k,S_i}, $\forall k \in \{1, 2, \ldots, K\}$.

Finally, the outage probability $\overline{\text{Pr}}_{\text{Out},i}$ can also be proved to be convex as it has a similar form to $\overline{\mathcal{M}}_{(i)}(z)$. Therefore, the proof of convexity for $\overline{\text{Pr}}_{\text{Out},i}$ is eliminated for brevity. ■

B. *Proof of Concavity*

With respect to the term $-\overline{\mathcal{M}}_{(i)}\left(\beta_m, \mathbf{P}_{R,S_i}\right)$ in the objective function of (20), and constraints (21b) and (24b), it is well-known that a function $f(x)$ is (strictly) convex if and only if $-f(x)$ is (strictly) concave [34]. Additionally, convexity/concavity is preserved under non-negative scaling and summation. Therefore, $-\overline{\mathcal{M}}_{(i)}\left(\beta_m, \mathbf{P}_{R,S_i}\right)$ is strictly concave, since it has already been proved that $\overline{\mathcal{M}}_{(i)}\left(\beta_m, \mathbf{P}_{R,S_i}\right)$ is strictly convex. ■

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